6.4: Values of the Trigonometric Functions

E. Kim

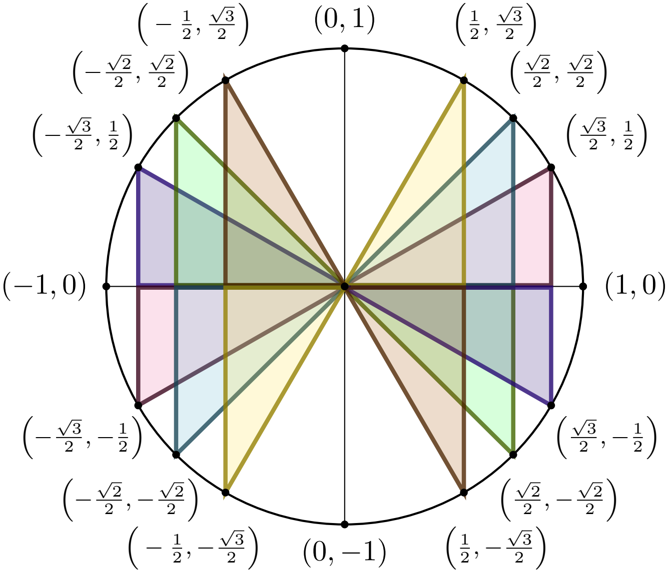
MTH 151

All notation and terminology is based on Swokowski, Cole. *Algebra and Trigonometry: with analytic geometry*. Classic 12th Edition.

Accompanying handout:

- Black-and-white: `http://www.uwlax.edu/faculty/ekim/resources/unit-circle.pdf`
- Color: `http://www.uwlax.edu/faculty/ekim/resources/unit-circle-color.pdf`

Goal



$\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$
$(0, 1)$
$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$
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$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$
$(-1, 0)$
$(1, 0)$
$\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$
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$(0, -1)$
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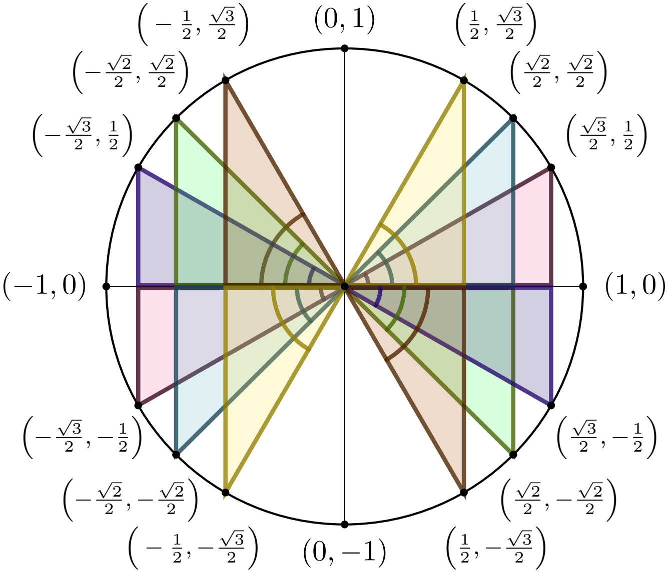

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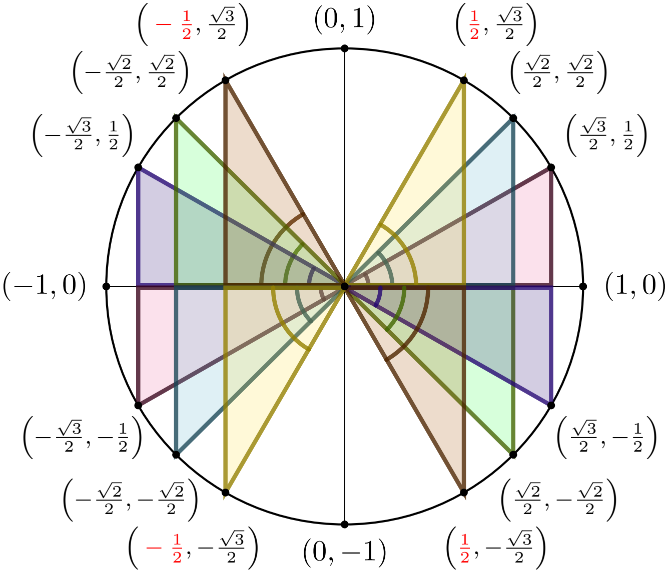

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Why understand the unit circle?

For all special angles, we can compute sine, cosine, etc. by knowing these values only for 30°, 45°, and 90°.

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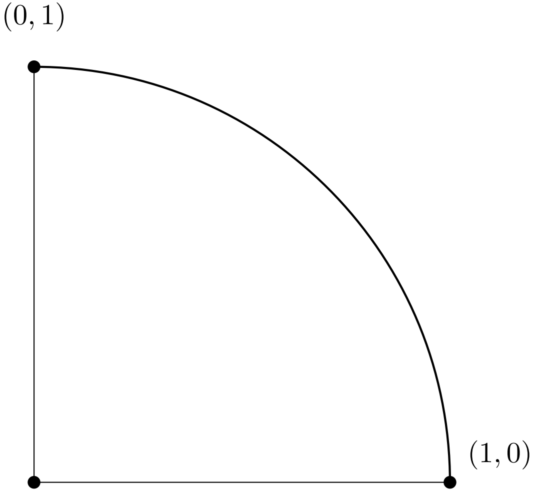
For all special angles, we can compute sine, cosine, etc. by knowing these values only for 30°, 45°, and 90°.

For the other special angles, just change the sign as appropriate.

The First Quadrant



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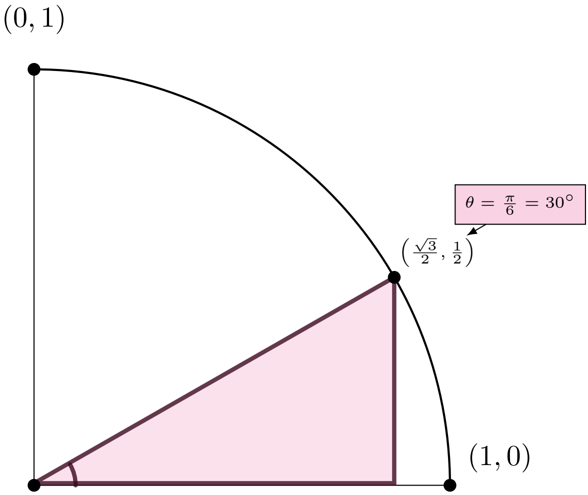

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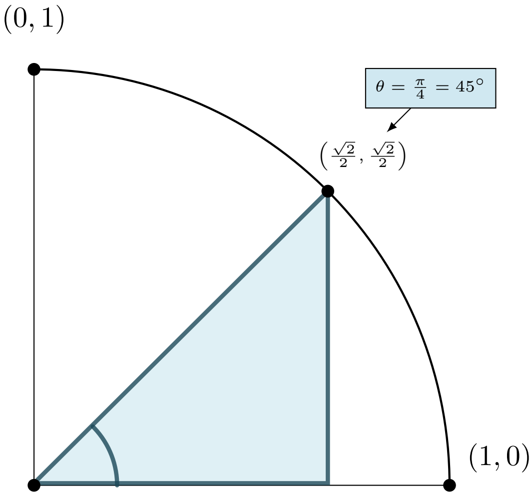

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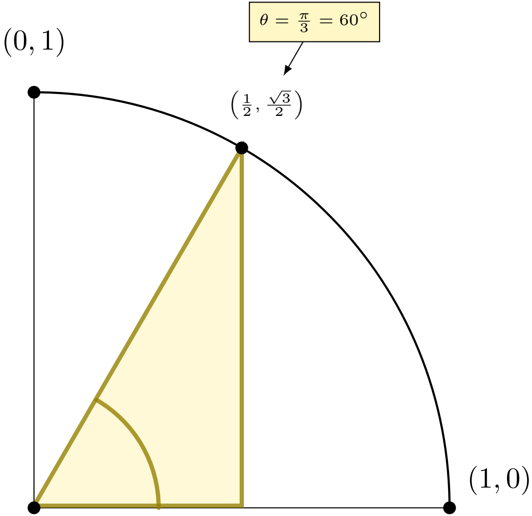

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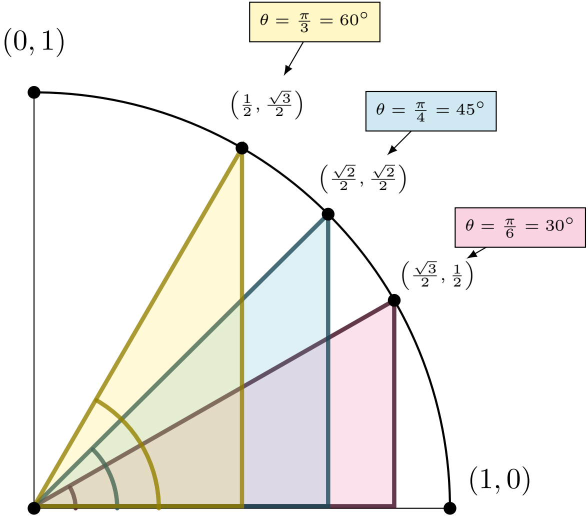
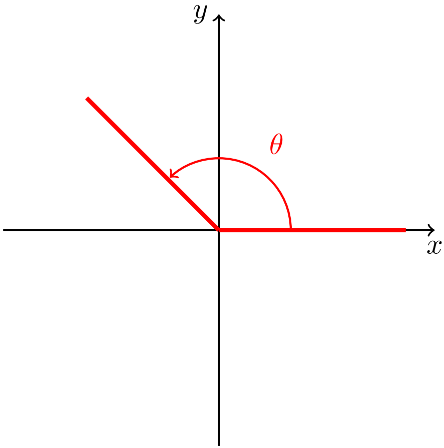


What is a reference angle?



Take a nonquadrantal[1] angle θ.

[1]Nonquadrantal means that θ is not a multiple of 90°.

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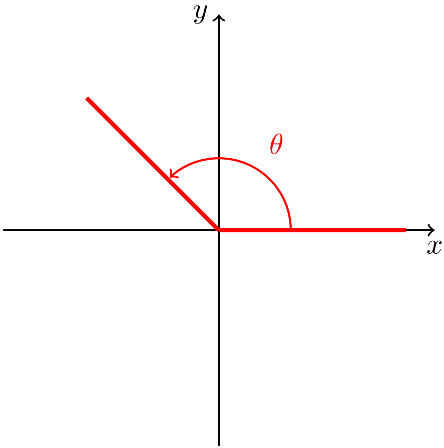

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θ_R, the reference angle

The acute angle made by

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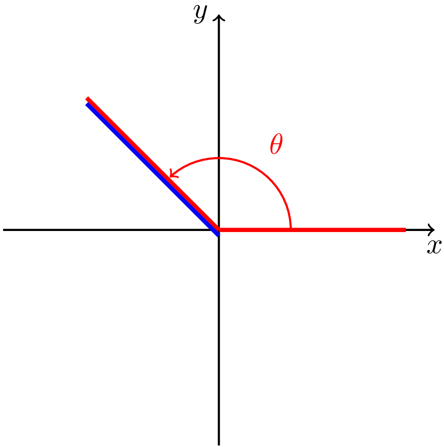

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The acute angle made by

- terminal side of θ

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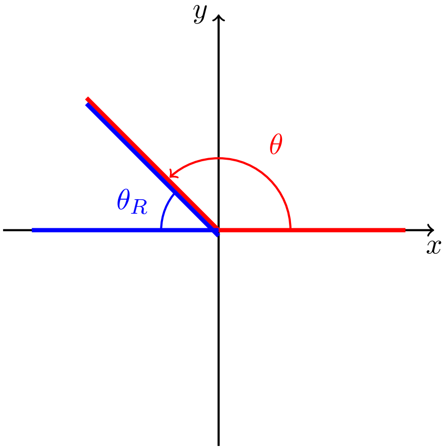

Take a nonquadrantal[1] angle θ.

θ_R, the reference angle

The acute angle made by

- terminal side of θ
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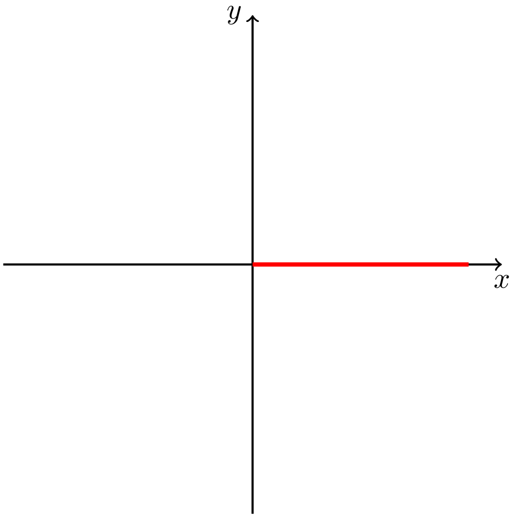
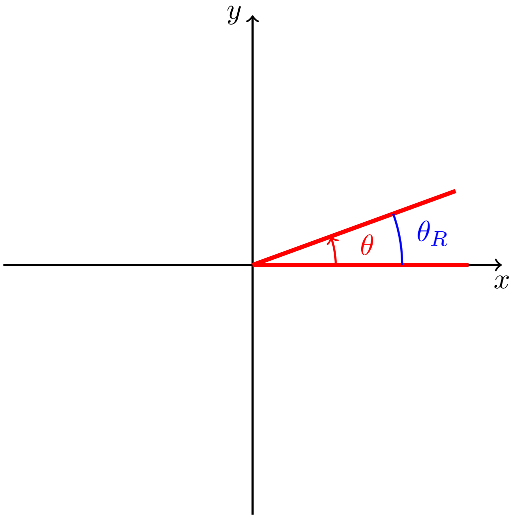
Formula for reference angle θ_R in every quadrant



Formula for reference angle θ_R in every quadrant

θ in Quadrant 1

$\theta_R = \theta$



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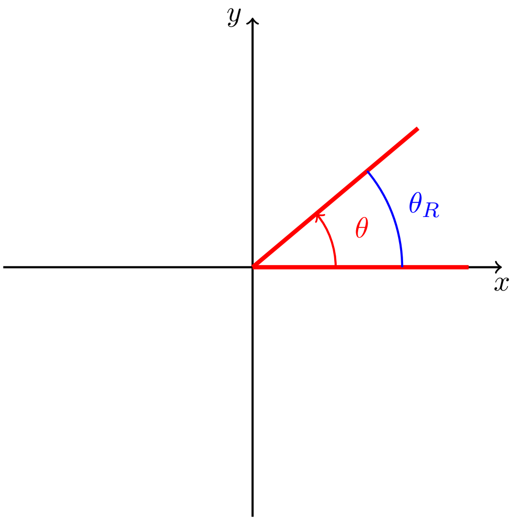

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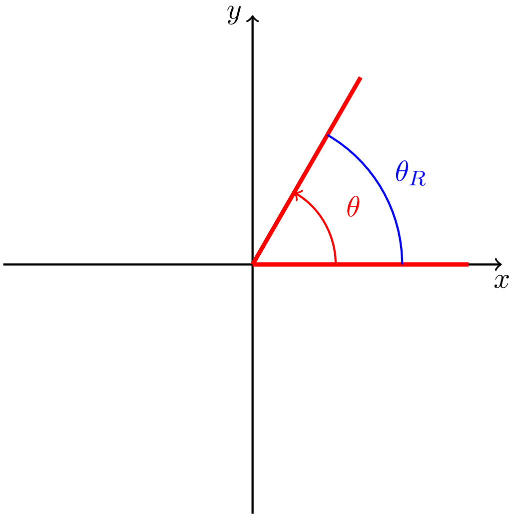

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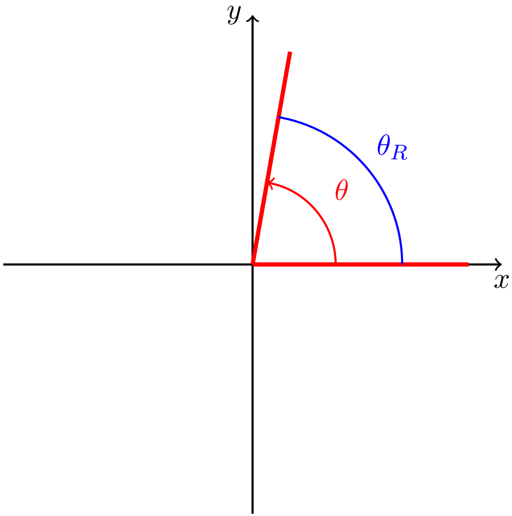

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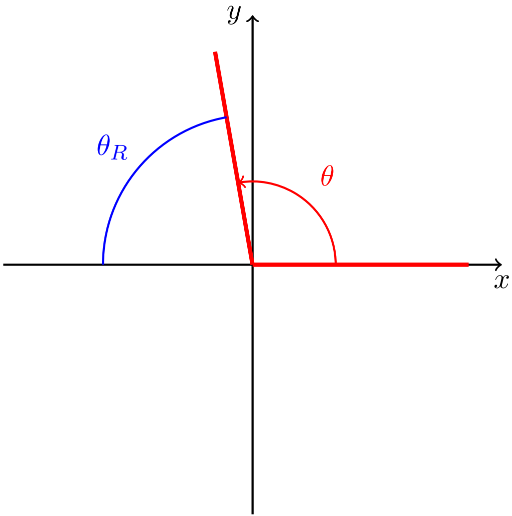

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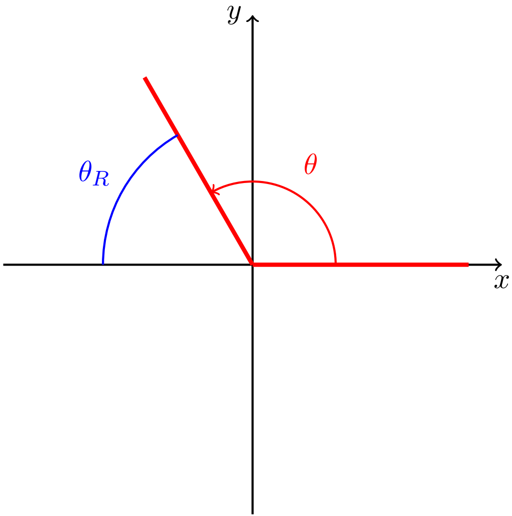

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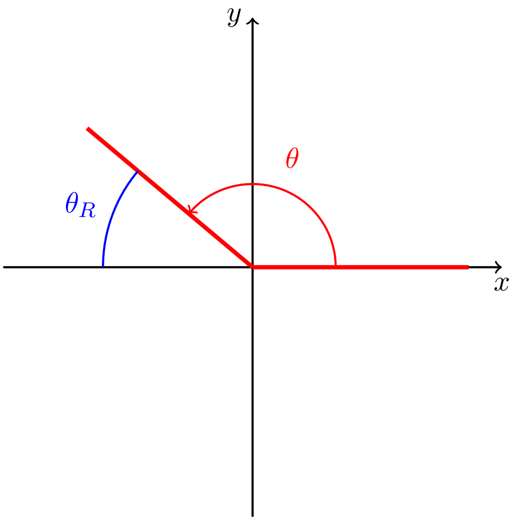

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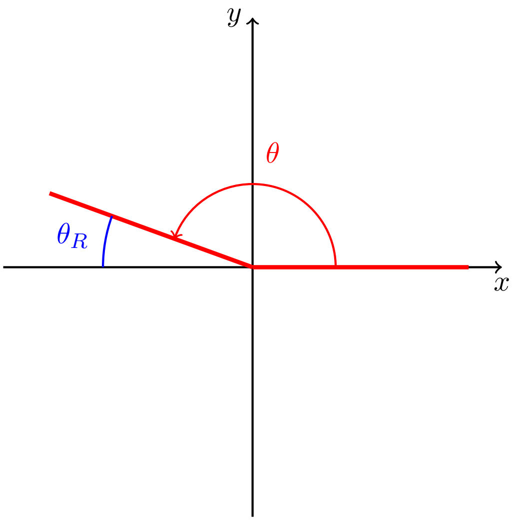

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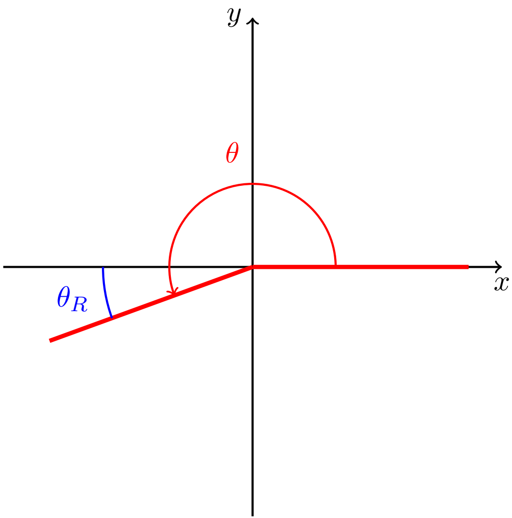

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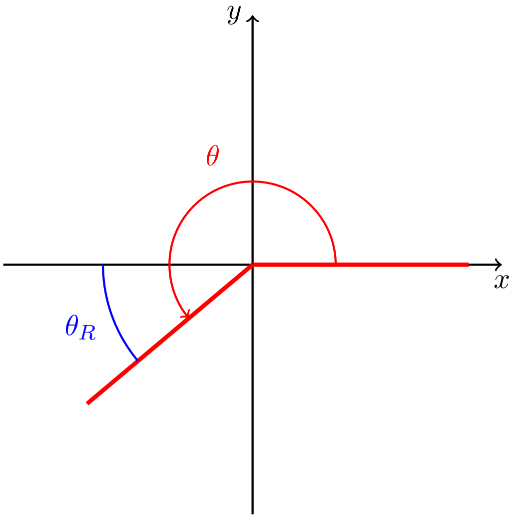

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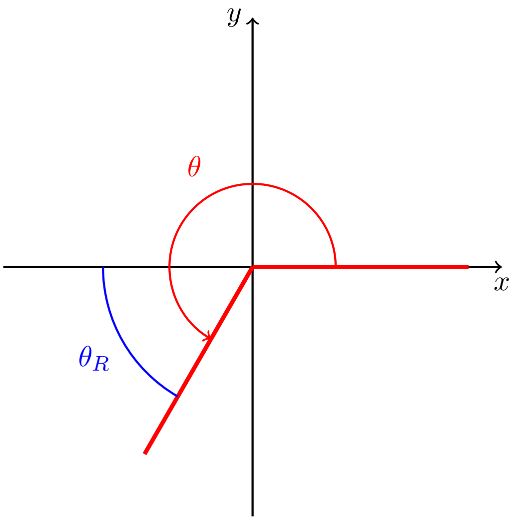

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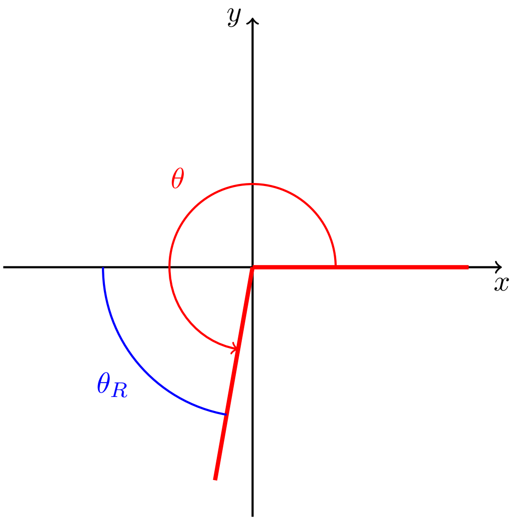

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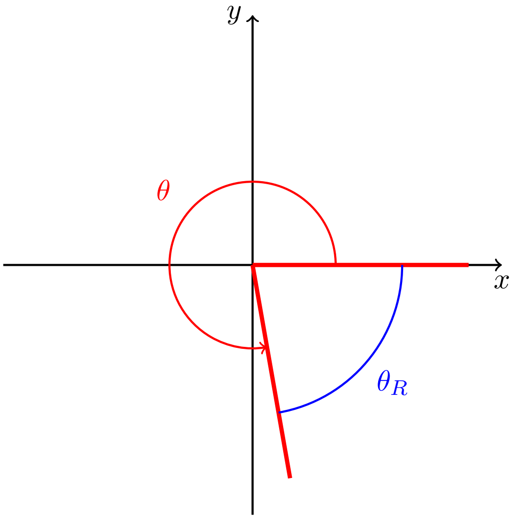

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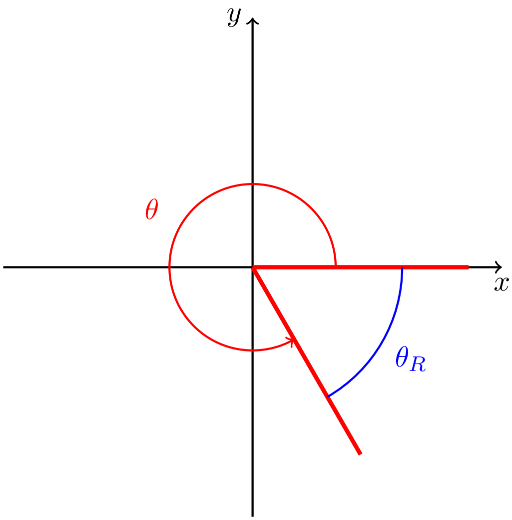

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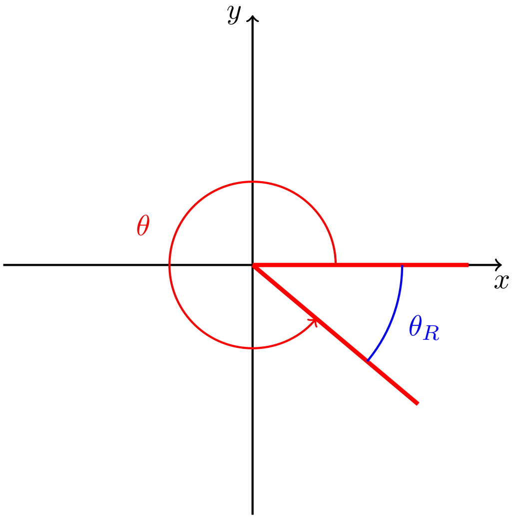

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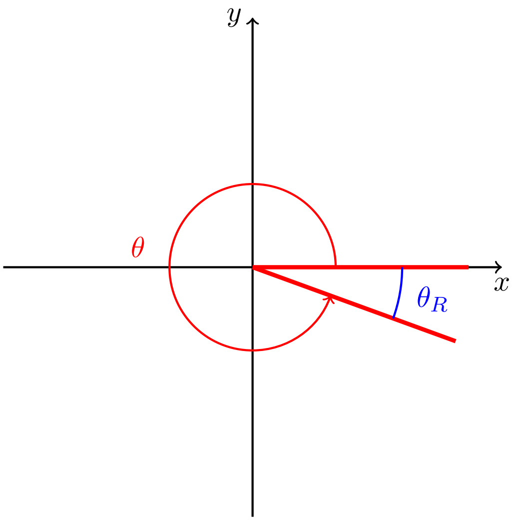
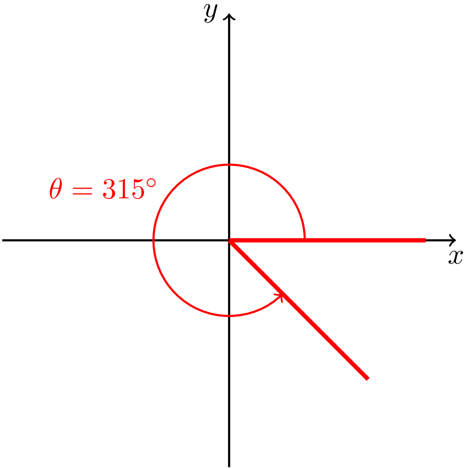


Example: $\theta = 315^\circ$



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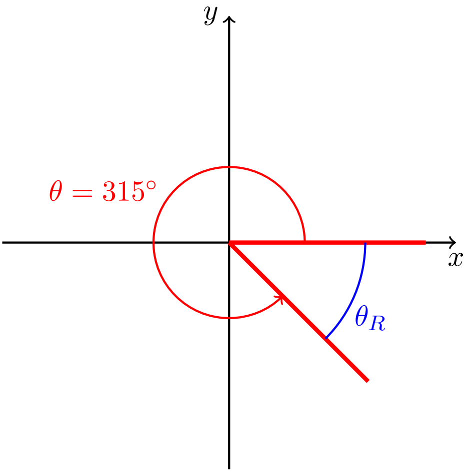

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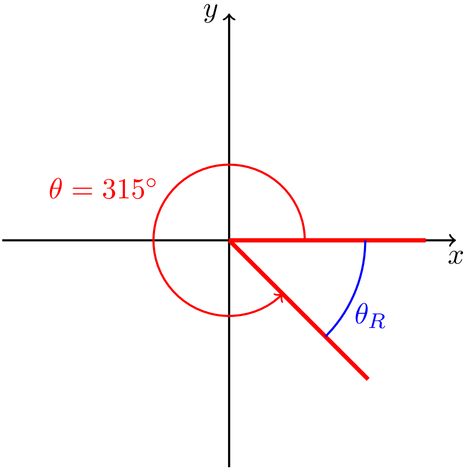
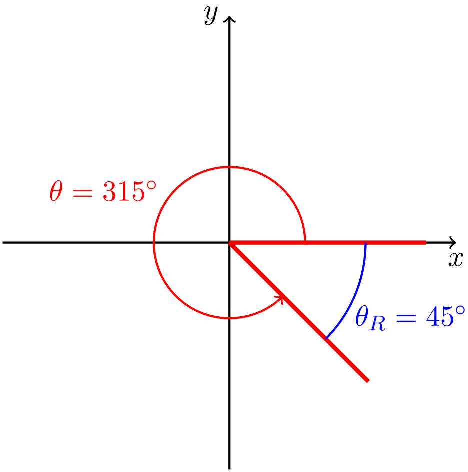


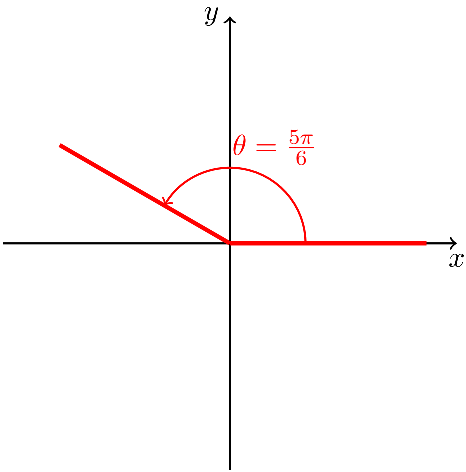
$\theta_R = 360^\circ - 315^\circ$

Example: $\theta = 315°$



$$\theta_R = 360° - 315° = 45°$$

Example: $\theta = \frac{5\pi}{6}$



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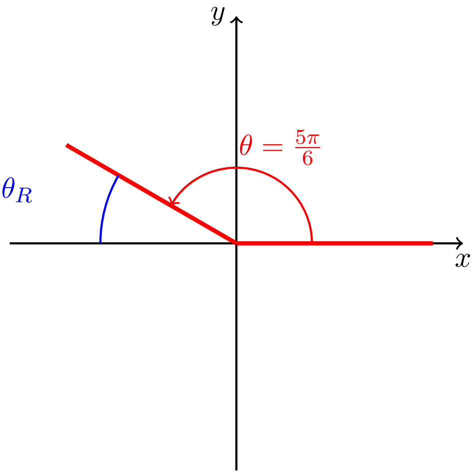

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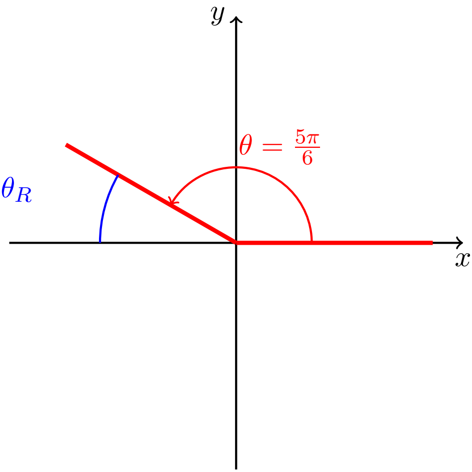
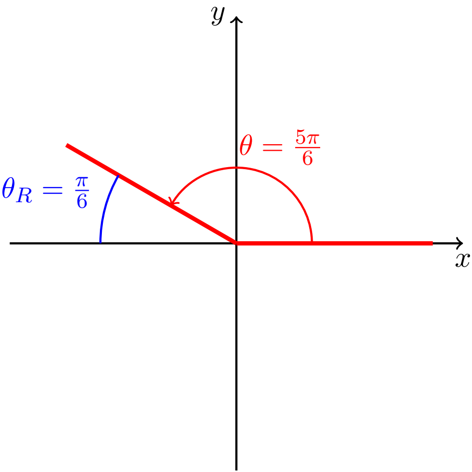


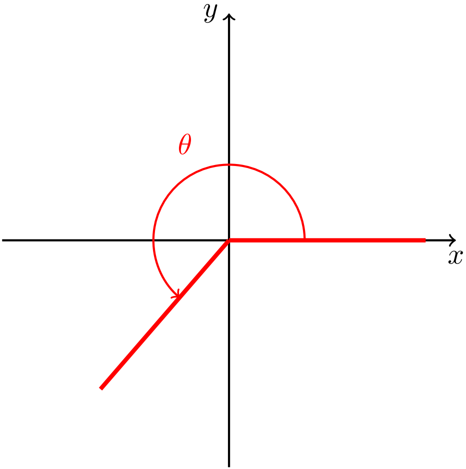
$\theta_R = \pi - \frac{5\pi}{6}$

Example: $\theta = \frac{5\pi}{6}$



$$\theta_R = \pi - \frac{5\pi}{6} = \frac{\pi}{6}$$

Example: $\theta = 4$. (Note this is four **radians**, not degrees!)



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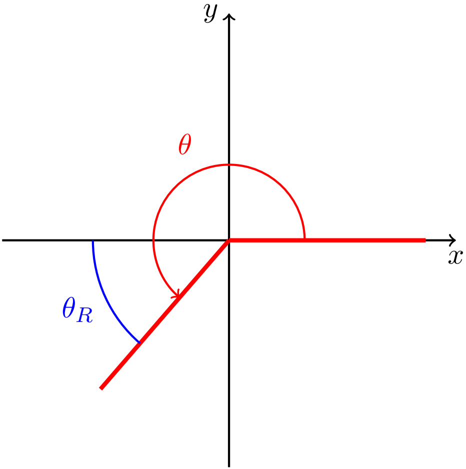

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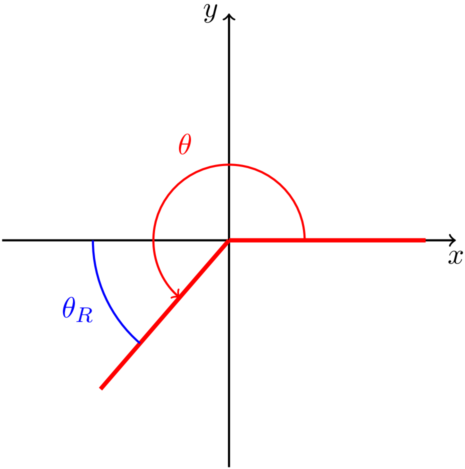

$\theta_R = 4 - \pi$

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Example: $\theta = -240^\circ$.

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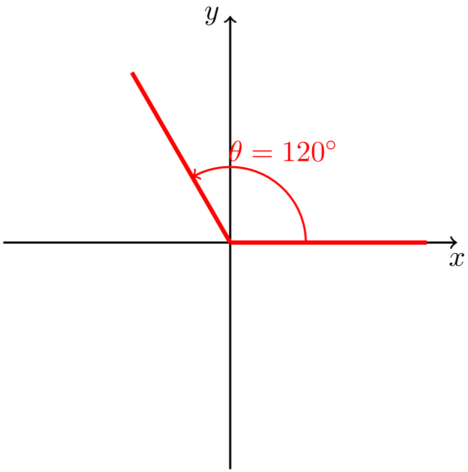
To find θ_R, use $\theta = 120^\circ$.

Example: $\theta = -240^\circ$, but using 120°



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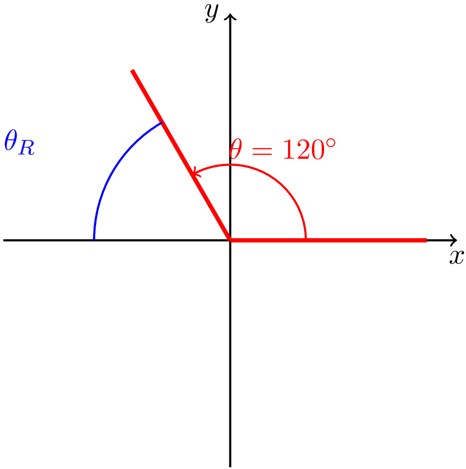

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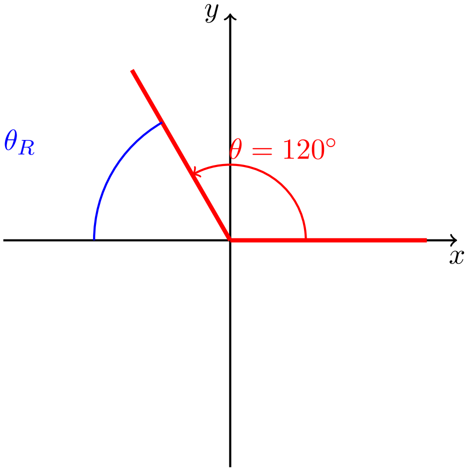
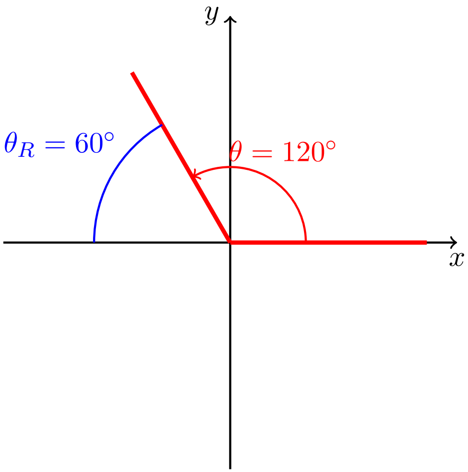


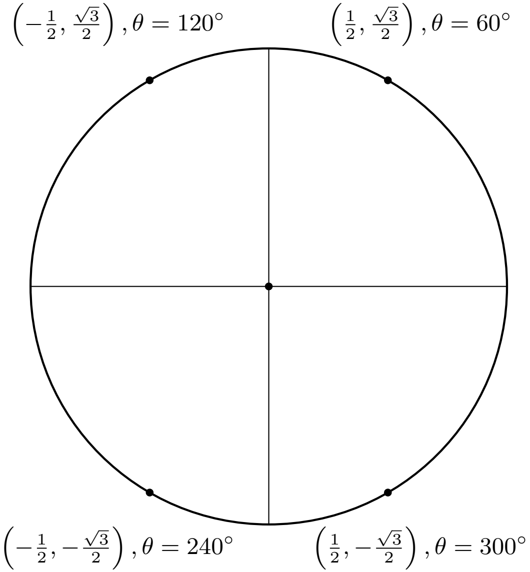
$$\theta_R = 180^\circ - 120^\circ$$

Example: $\theta = -240°$, but using $120°$

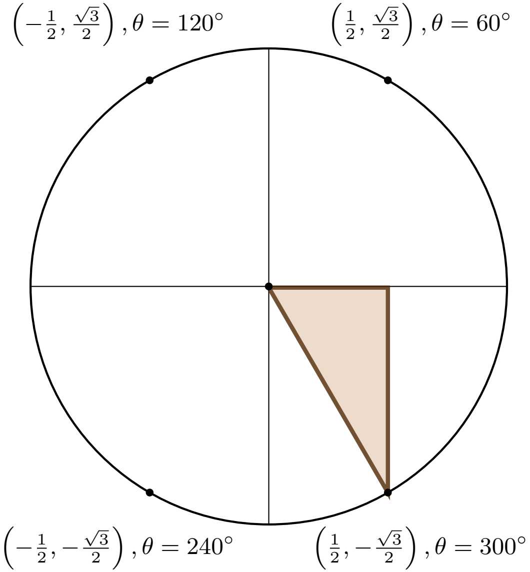


$$\theta_R = 180° - 120° = 60°$$

Reference angles and exact values of $\sin$, $\cos$, and $\tan$



Reference angles and exact values of $\sin$, $\cos$, and $\tan$



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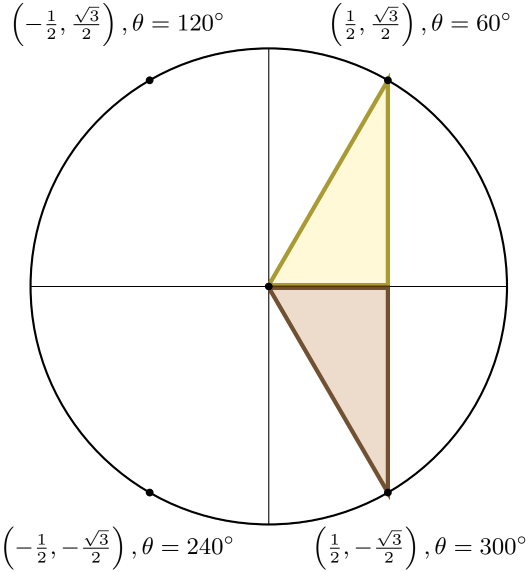

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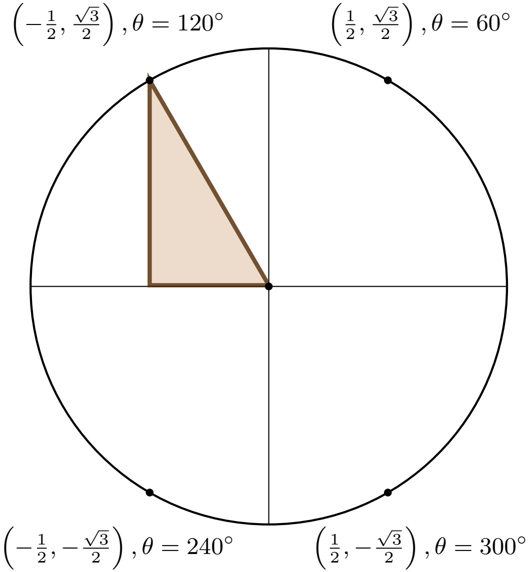

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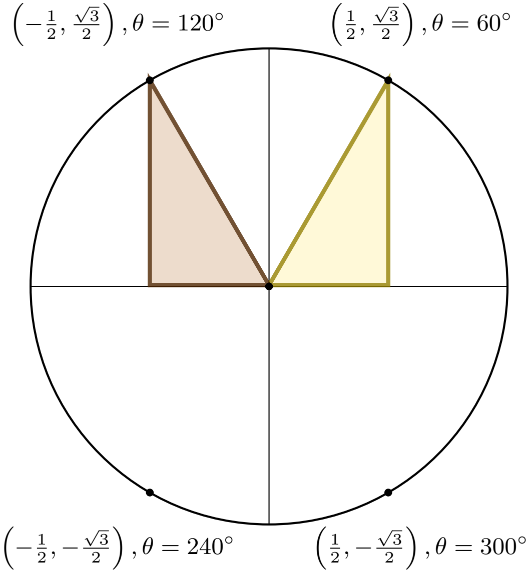

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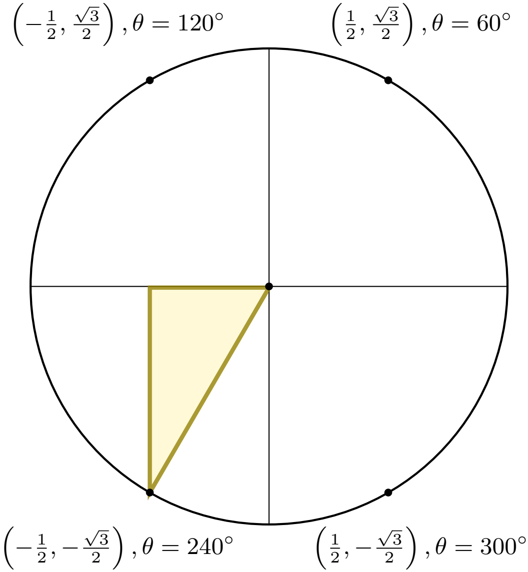

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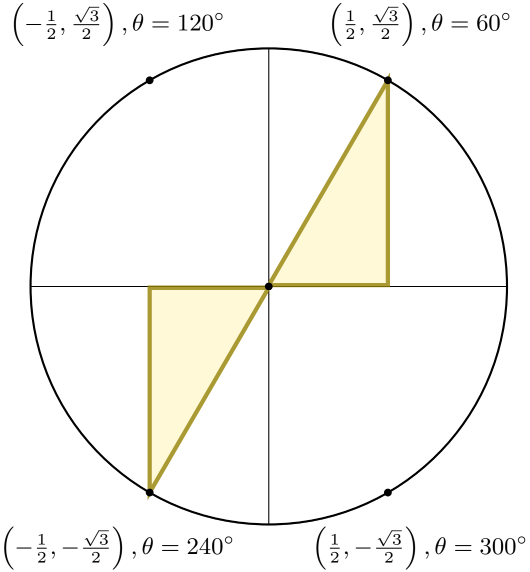

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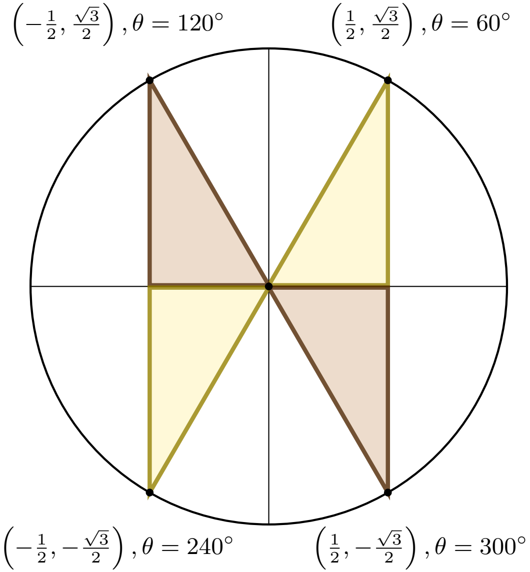

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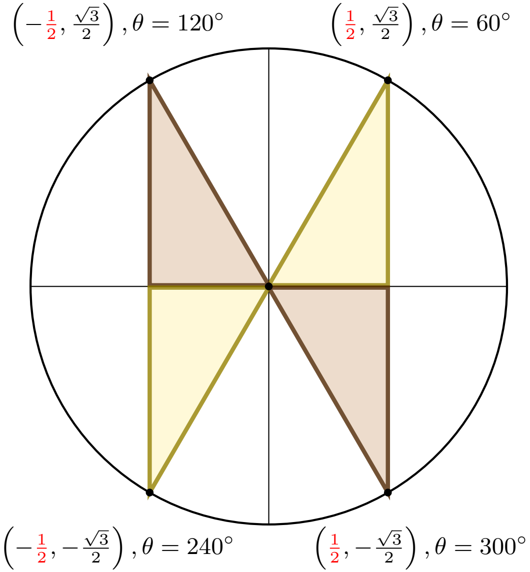

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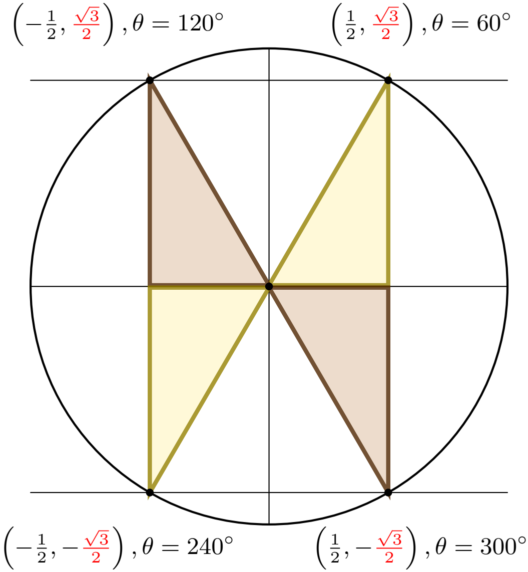
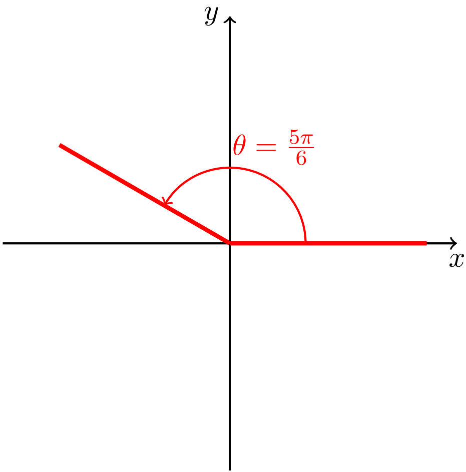


If $\theta = \frac{5\pi}{6}$, find $\sin\theta$, $\cos\theta$, and $\tan\theta$



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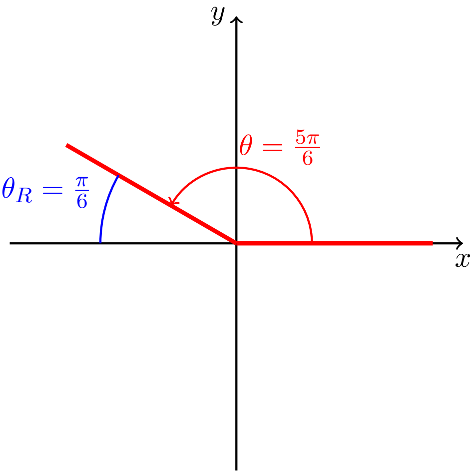

$$\theta_R = \pi - \frac{5\pi}{6} = \frac{\pi}{6}$$

If $\theta = \frac{5\pi}{6}$, find $\sin\theta$, $\cos\theta$, and $\tan\theta$

Reference angle is $\theta_R = \frac{\pi}{6}$

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$$\sin\frac{\pi}{6} = \frac{1}{2} \qquad \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2} \qquad \tan\frac{\pi}{6} = \frac{\sqrt{3}}{3}$$

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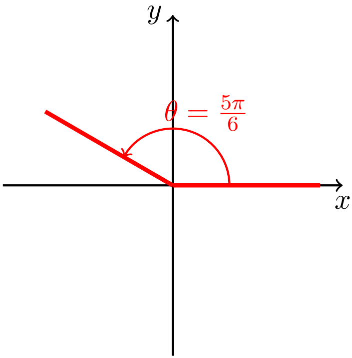

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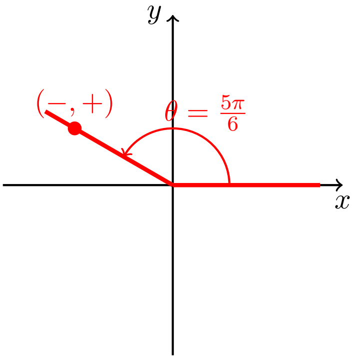

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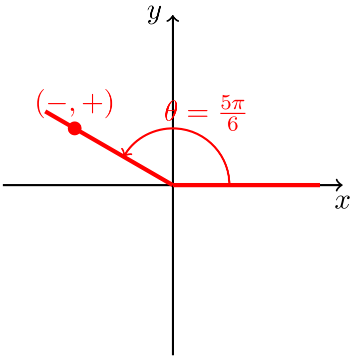
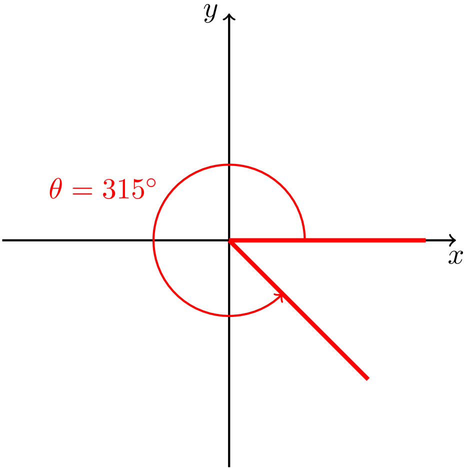


$$\sin\frac{5\pi}{6} = +\frac{1}{2} \qquad \cos\frac{5\pi}{6} = -\frac{\sqrt{3}}{2} \qquad \tan\frac{5\pi}{6} = -\frac{\sqrt{3}}{3}$$

If $\theta = 315^\circ$, find $\sin\theta$, $\cos\theta$, and $\tan\theta$



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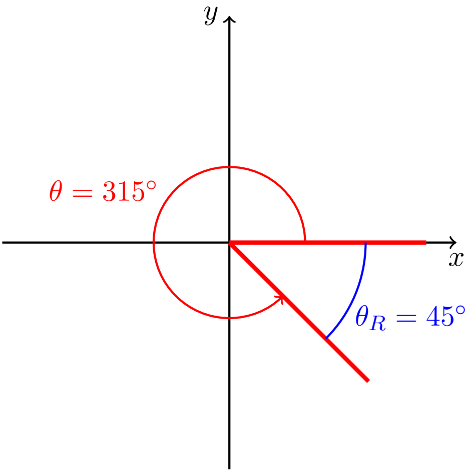

$$\theta_R = 360^\circ - 315^\circ = 45^\circ$$

If $\theta = 315^\circ$, find $\sin\theta$, $\cos\theta$, and $\tan\theta$

Reference angle is $\theta_R = 45^\circ$

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Reference angle is $\theta_R = 45^\circ$

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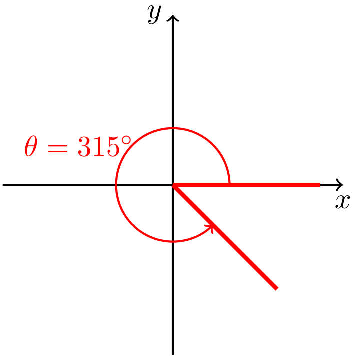

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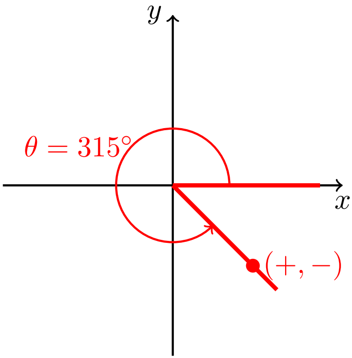

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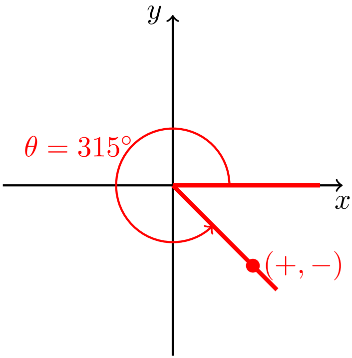

$$\sin 315^\circ = -\frac{\sqrt{2}}{2} \qquad \cos 315^\circ = +\frac{\sqrt{2}}{2} \qquad \tan 315^\circ = -1$$

Summary of using reference angles

1. Find the reference angle θ_R for your angle θ.
2. Compute $\sin$, $\cos$, and $\tan$ for the reference angle θ_R.
3. Adjust the sign based on the quadrant of terminal side of θ.

Finding angles with a calculator

Inverse trigonometric functions

Problem

If θ is an acute angle and $\sin\theta = 0.6635$, what is θ?

Inverse trigonometric functions

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If $\sin\theta = k$, then $\theta = \sin^{-1} k$.

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$$\theta = \sin^{-1}(0.6635)$$

Inverse trigonometric functions

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If θ is an acute angle and $\sin\theta = 0.6635$, what is θ?

If $\sin\theta = k$, then $\theta = \sin^{-1} k$.

$$\theta = \sin^{-1}(0.6635) \approx 41.57^\circ \approx 0.7255$$

Inverse trigonometric functions

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$$\theta = \sin^{-1}(0.6635) \approx \underbrace{41.57^\circ}_{\text{degrees}} \approx \underbrace{0.7255}_{\text{radians}}$$

If $\cos\theta = k$, then $\theta = \cos^{-1} k$.
If $\tan\theta = k$, then $\theta = \tan^{-1} k$.

What about $\csc$, $\sec$, and $\cot$?

Use the reciprocal formulas:

$$\csc\theta = \frac{1}{\sin\theta} \quad \sec\theta = \frac{1}{\cos\theta} \quad \cot\theta = \frac{1}{\tan\theta}$$

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Example: $\csc\theta = 2$

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Convert to

$$\frac{1}{\sin\theta} = 2$$

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Convert to

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Take reciprocal of both sides

$$\sin\theta = \frac{1}{2}$$

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Use inverse sine function (also called **arcsin** or **asin**)

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$$\frac{1}{\sin\theta} = 2$$

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Use inverse sine function (also called **arcsin** or **asin**)

$$\theta = \sin^{-1}\left(\frac{1}{2}\right) = 30^\circ$$

An example of each inverse trig function

$\sin\theta = 0.5$

$\theta = \sin^{-1}(\frac{1}{2}) = 30^\circ \approx 0.5236$

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$\theta = \tan^{-1}(\frac{1}{2}) \approx 26.57^\circ \approx 0.4636$

Column on right copies column on left
because of reciprocal identites

Since $f(x) = \sin(x)$ is periodic, what is $\sin^{-1} k$ giving you?

Inverse Sine

If you put $\sin^{-1} k$ into your calculator, the answer will be an angle

- in $[-\frac{\pi}{2}, \frac{\pi}{2}]$ using radian mode
- in $[-90^\circ, 90^\circ]$ in degree mode

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Inverse Cosine

If you put $\cos^{-1} k$ into your calculator, the answer will be an angle

- in $[0, \pi]$ using radian mode
- in $[0, 180°]$ in degree mode

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Inverse Sine

If you put $\sin^{-1} k$ into your calculator, the answer will be an angle

- in $[-\frac{\pi}{2}, \frac{\pi}{2}]$ using radian mode
- in $[-90^\circ, 90^\circ]$ in degree mode

Inverse Cosine

If you put $\cos^{-1} k$ into your calculator, the answer will be an angle

- in $[0, \pi]$ using radian mode
- in $[0, 180^\circ]$ in degree mode

Inverse Tangent

If you put $\tan^{-1} k$ into your calculator, the answer will be an angle

- in $(-\frac{\pi}{2}, \frac{\pi}{2})$ using radian mode
- in $(-90^\circ, 90^\circ)$ in degree mode

Examples with negative values

$\sin\theta = -0.5$

$\theta = \sin^{-1}(-\frac{1}{2}) = -30^\circ \approx -0.5236$

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$$\theta = \sin^{-1}(-\tfrac{1}{2}) = -30^\circ \approx -0.5236$$

$\cos\theta = -0.5$

$$\theta = \cos^{-1}(-\tfrac{1}{2}) = 120^\circ \approx 2.0944$$

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$\tan\theta = -0.5$

$\theta = \tan^{-1}(-\frac{1}{2}) \approx -26.57^\circ \approx -0.4636$

If the calculator doesn't give you the angle θ you wanted...

...use reference angles to find the angle you want!

Find θ such that $\tan\theta = -0.4623$ and $0^\circ \leq \theta < 360^\circ$

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Putting $\tan^{-1}(-0.4623)$ in the calculator (in degree mode).

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- Get $\approx -24.8^\circ$.

Find θ such that $\tan\theta = -0.4623$ and $0^\circ \leq \theta < 360^\circ$

Putting $\tan^{-1}(-0.4623)$ in the calculator (in degree mode).

- Get $\approx -24.8^\circ$.

Problem: We wanted θ such that $0^\circ \leq \theta < 360^\circ$.

Find θ such that $\tan\theta = -0.4623$ and $0^\circ \leq \theta < 360^\circ$

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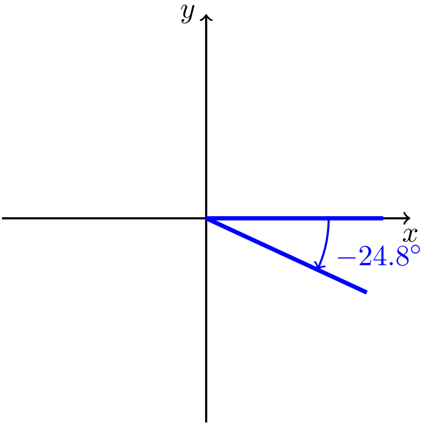

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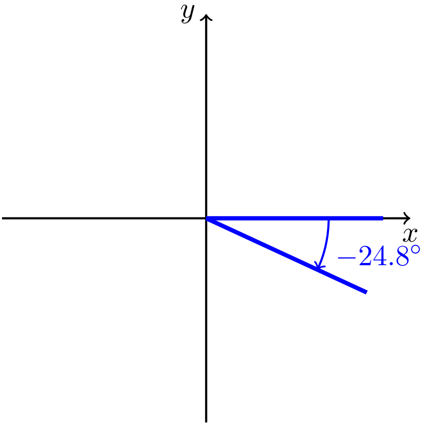

Solution: $\tan$ is 180°-periodic:

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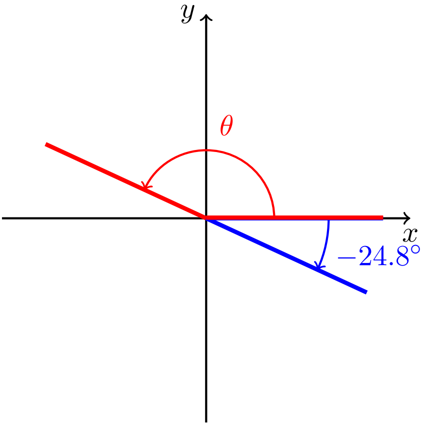


Solution: $\tan$ is 180°-periodic:

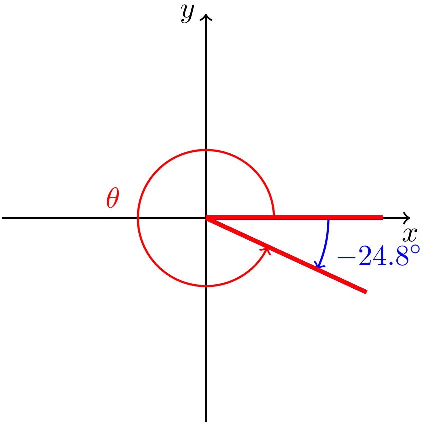
- Add 180° to -24.8°
  - $\theta = 155.2^\circ$

Find θ such that $\tan\theta = -0.4623$ and $0^\circ \le \theta < 360^\circ$

Putting $\tan^{-1}(-0.4623)$ in the calculator (in degree mode).

- Get $\approx -24.8^\circ$.

Problem: We wanted θ such that $0^\circ \le \theta < 360^\circ$.



Solution: $\tan$ is 180°-periodic:

- Add 180° to -24.8°
  - $\theta = 155.2^\circ$
- Add 180° to 155.2°
  - $\theta = 335.2^\circ$

Find θ such that $\cos\theta = -0.3842$ and $0 \leq \theta < 2\pi$

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Putting $\cos^{-1}(-0.3842)$ in the calculator (in radian mode).

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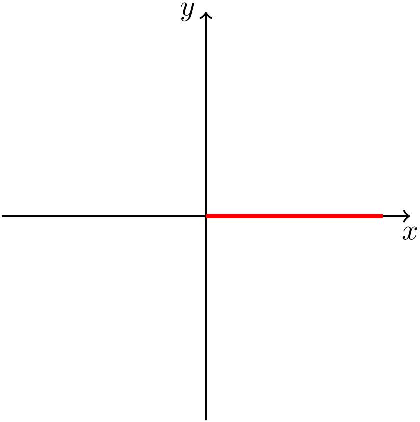
Putting $\cos^{-1}(-0.3842)$ in the calculator (in radian mode).

- Get ≈ 1.9651.

Find θ such that $\cos\theta = -0.3842$ and $0 \leq \theta < 2\pi$

Putting $\cos^{-1}(-0.3842)$ in the calculator (in radian mode).

- Get ≈ 1.9651.
- Since 1.9651 is between 0 and π, reference angle is $\approx \pi - 1.9651 \approx 1.1765$



Find θ such that $\cos\theta = -0.3842$ and $0 \leq \theta < 2\pi$

Putting $\cos^{-1}(-0.3842)$ in the calculator (in radian mode).

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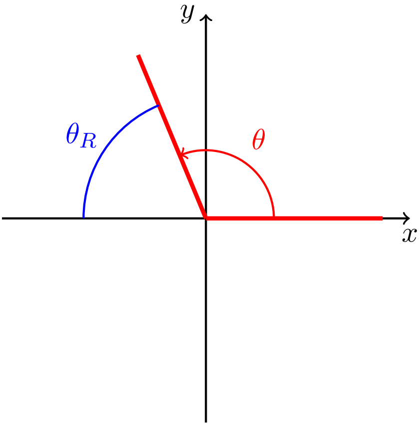

$\theta \approx 1.9651$ is a solution.

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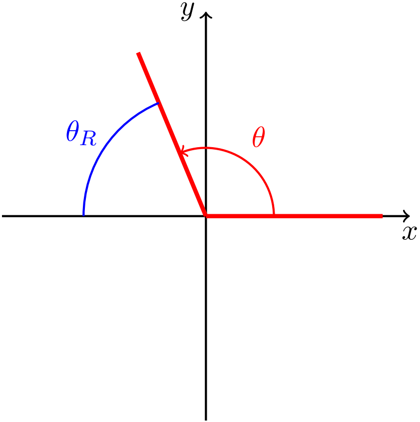

$\theta \approx 1.9651$ is a solution.

Find another θ with the same x value?

Find θ such that $\cos\theta = -0.3842$ and $0 \leq \theta < 2\pi$

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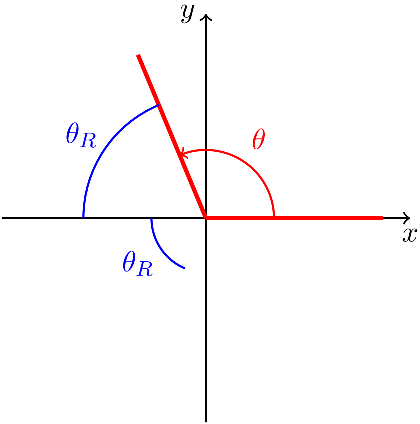

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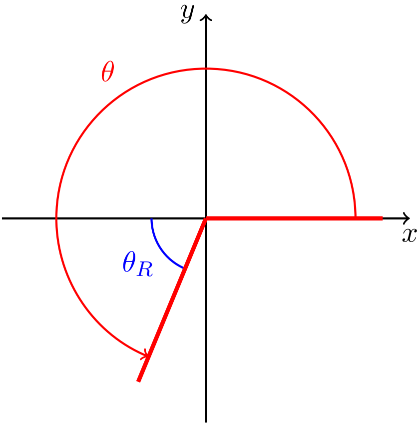

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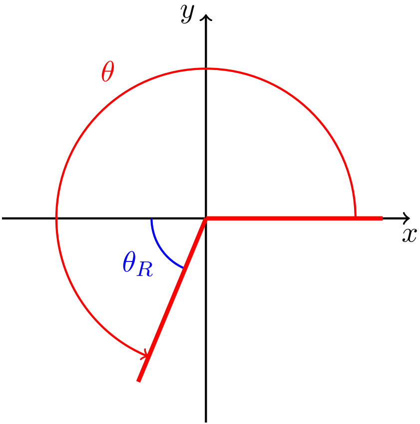


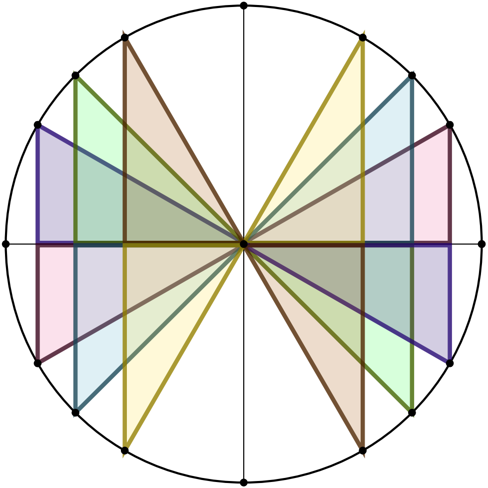
$\theta \approx 1.9651$ is a solution.

Find another θ with the same x value?

$\theta \approx \pi + 1.1765 \approx 4.3180$ is a second solution

Main idea

Use the symmetry in the circle with $\pm$ to get $\sin$, $\cos$, $\tan$



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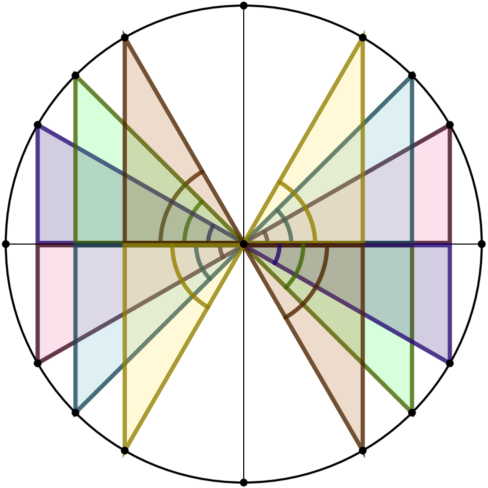
Use the symmetry in the circle with $\pm$ to get $\sin$, $\cos$, $\tan$



The angles which have related x and y value have the same reference angle!