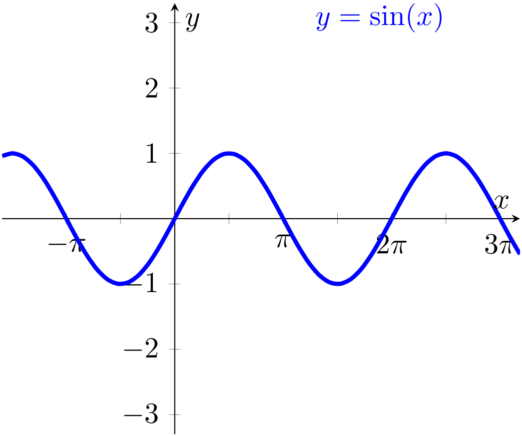
6.5: Trigonometric Graphs

E. Kim

MTH 151

All notation and terminology is based on Swokowski, Cole. *Algebra and Trigonometry: with analytic geometry*. Classic 12th Edition.

In Section 6.3, we graphed



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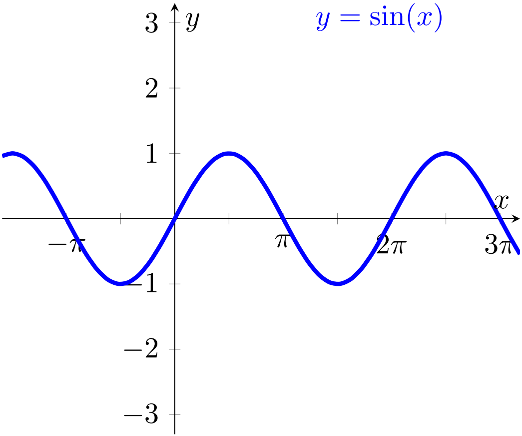
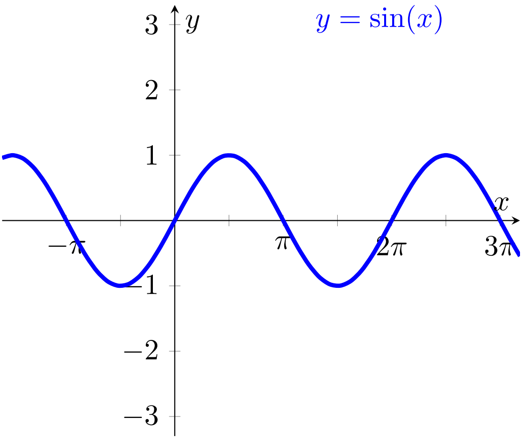


What does the graph of $y = a\sin(bx + c)$ look like?

Review $y = \sin(x)$



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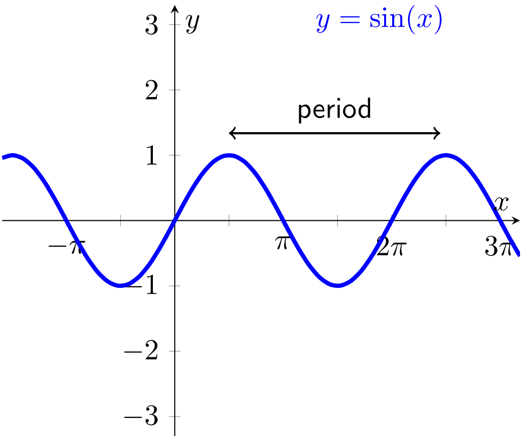
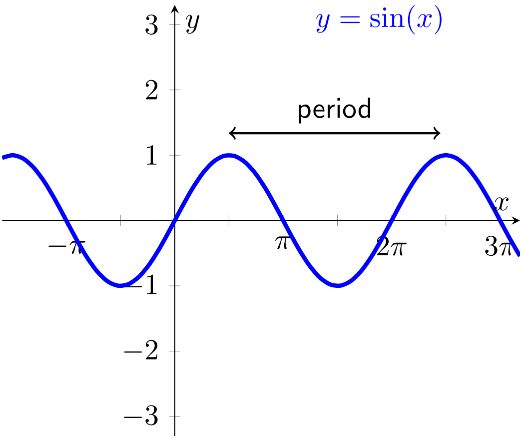


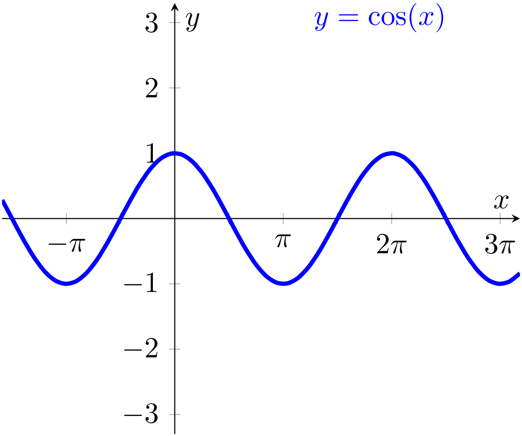
- period $= 2\pi$

Review $y = \sin(x)$



- period $= 2\pi$
- y-intercept at 0

Review $y = \cos(x)$



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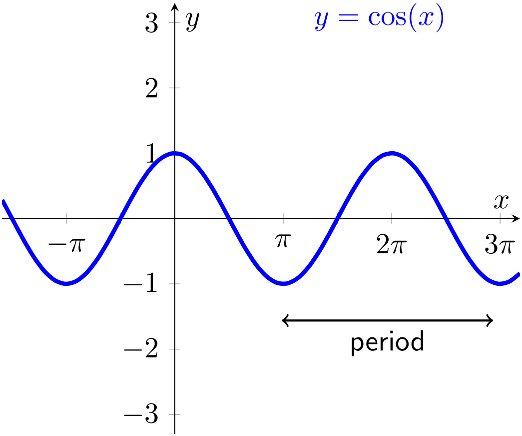

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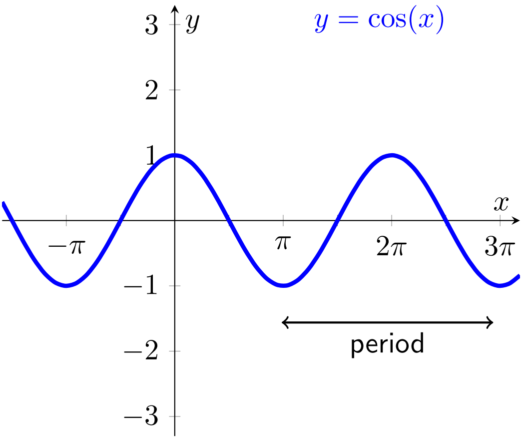
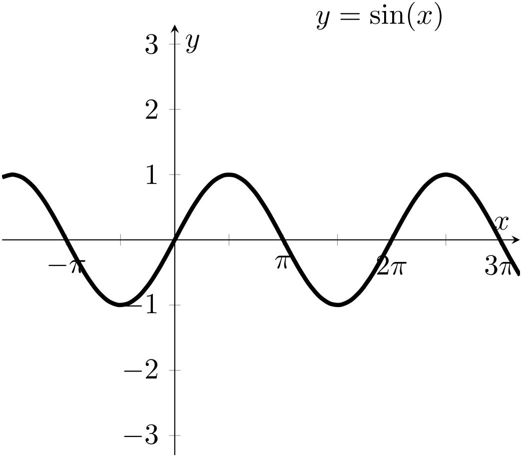


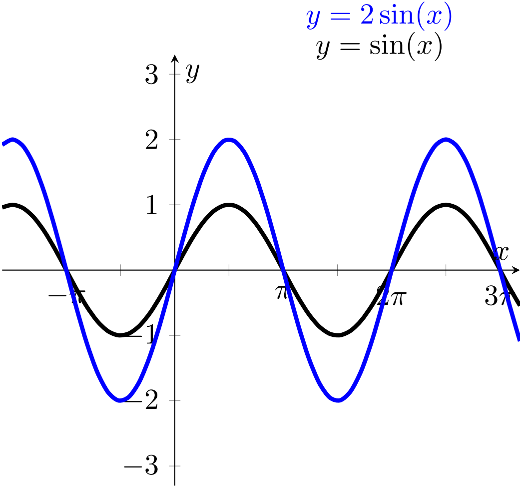
- period $= 2\pi$
- y-intercept at 1

Graph of $y = a\sin(x)$

$y = \sin(x)$ vs. $y = 2\sin(x)$



$y = \sin(x)$ vs. $y = 2\sin(x)$



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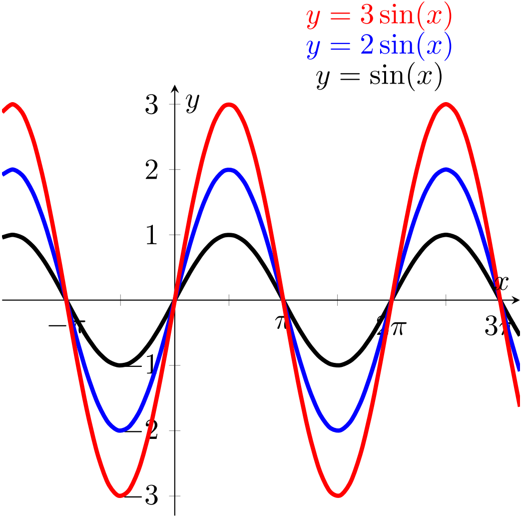
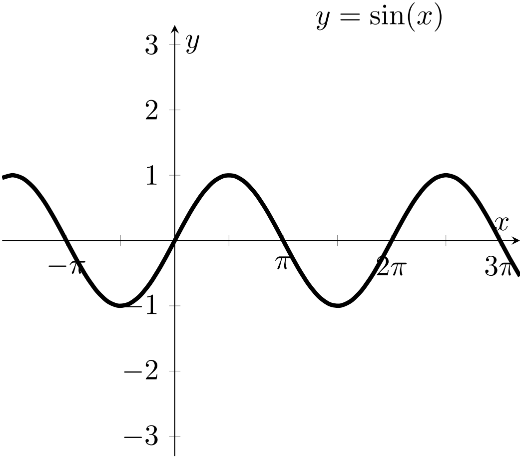


$y = \sin(x)$ vs. $y = \frac{1}{2}\sin(x)$



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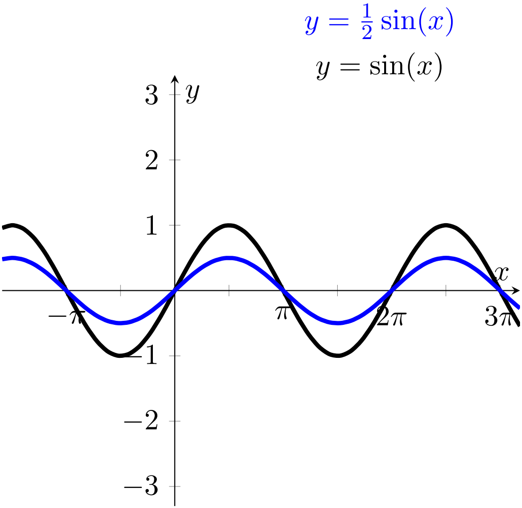

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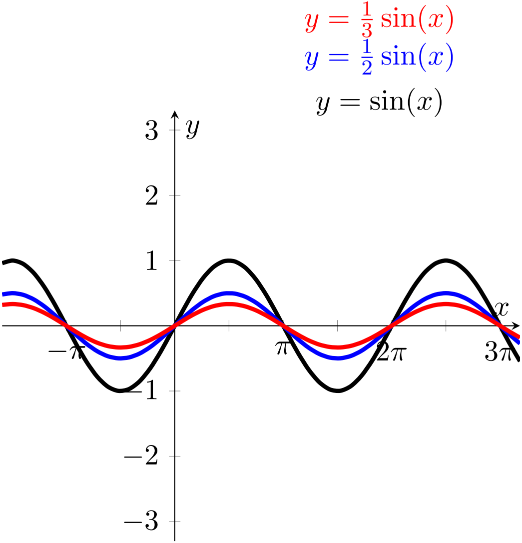
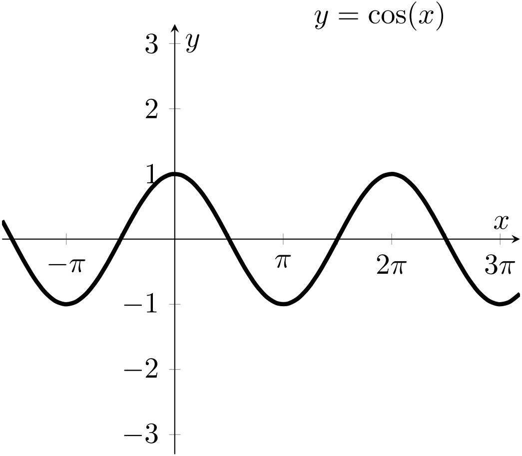


Is anything different for cosine?



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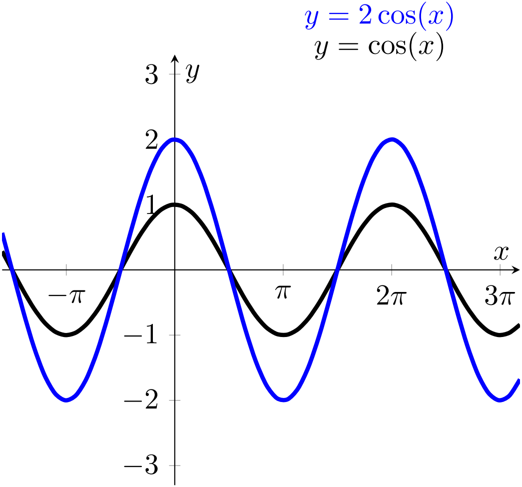

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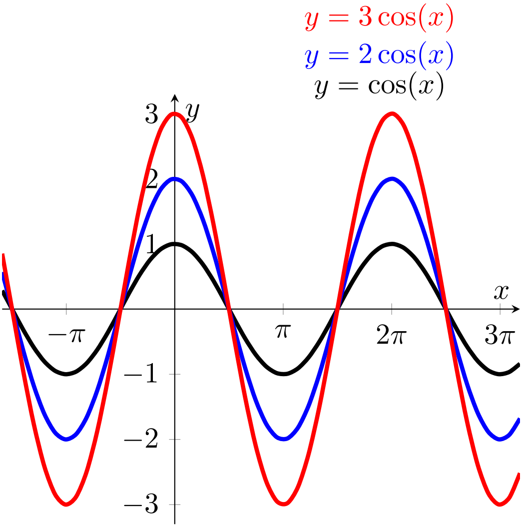


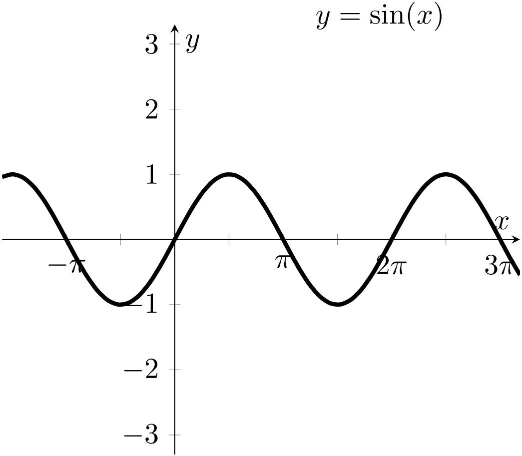
If $a > 0$, what does $y = a\sin(x)$ do?

Take home message

Stretches the graph vertically by a factor of a

- $a > 1 \leftrightsquigarrow$ expansion
- $a < 1 \leftrightsquigarrow$ contraction

$y = \sin(x)$ vs. $y = -\sin(x)$



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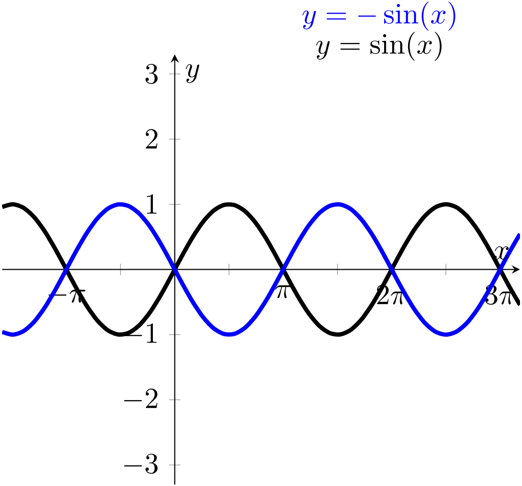

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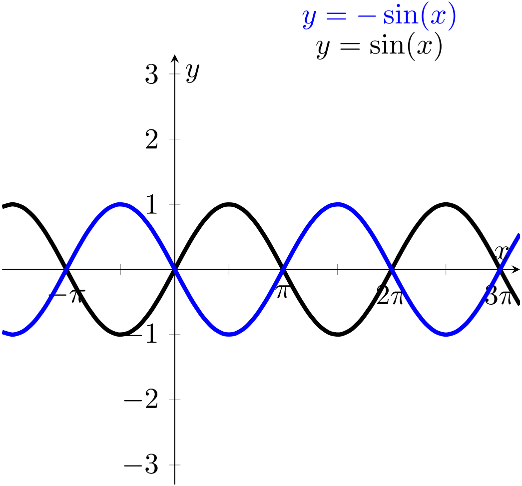
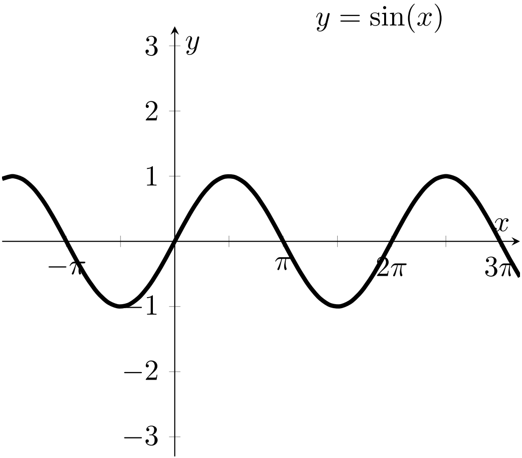


The minus sign in front of sine...

"flips" the graph across the x-axis

$y = \sin(x)$ vs. $y = -\sin(x)$



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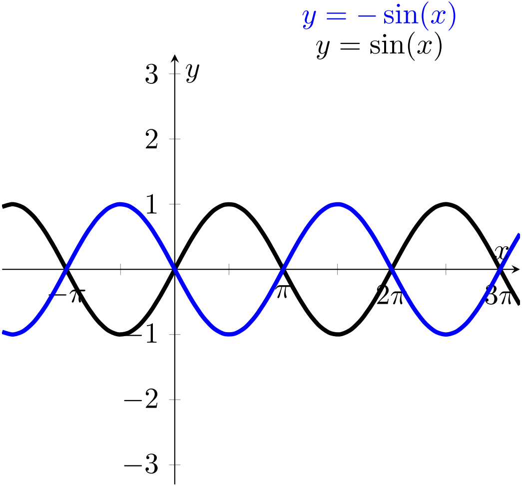

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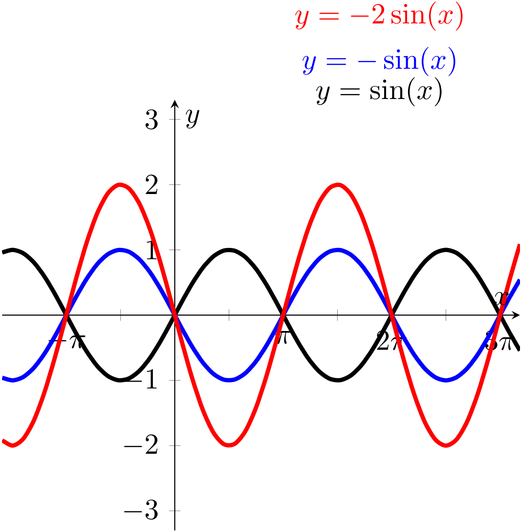

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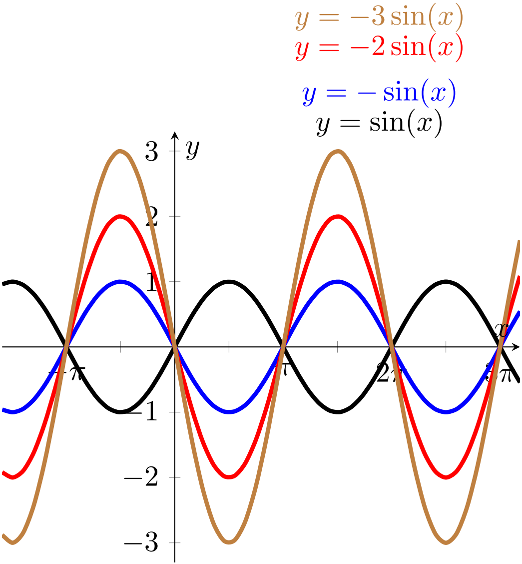
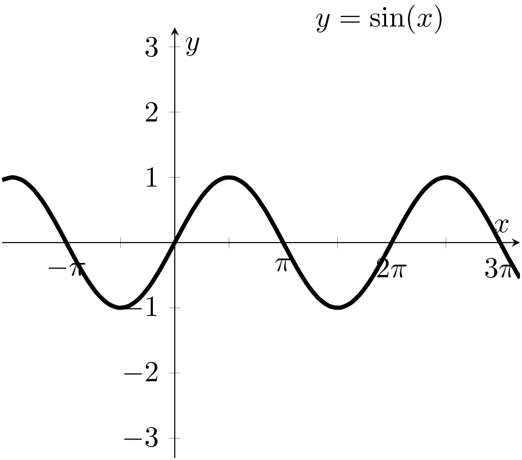


$y = \sin(x)$ vs. $y = -\frac{1}{2}\sin(x)$



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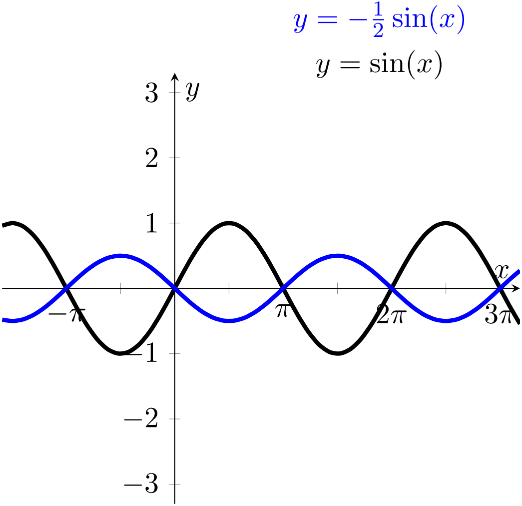

$y = a\sin(x)$ or $y = a\cos(x)$

Summary

$y = a\sin(x)$ or $y = a\cos(x)$

Summary

- The sign of $a \leftrightsquigarrow$ is graph flipped vertically?

$y = a\sin(x)$ or $y = a\cos(x)$

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- The sign of $a \leftrightsquigarrow$ is graph flipped vertically?
- The absolute value of a, namely $|a| \leftrightsquigarrow$ amount of vertical stretching

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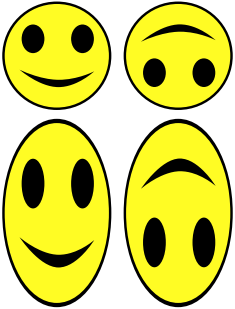

$y = a\sin(x)$ or $y = a\cos(x)$

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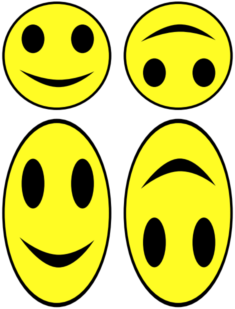
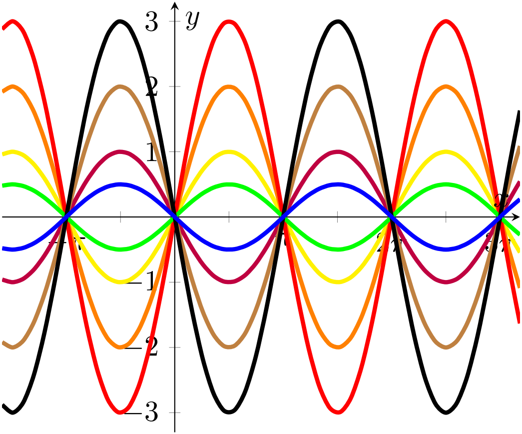


Definition

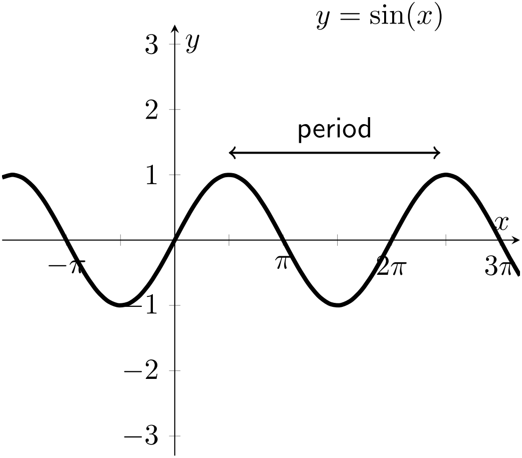
For $y = a\sin(x)$ or $y = a\cos(x)$, the number $|a|$ is the **amplitude**.

Summary with no words



Graph of $y = \sin(bx)$

$y = \sin(x)$ vs. $y = \sin(2x)$



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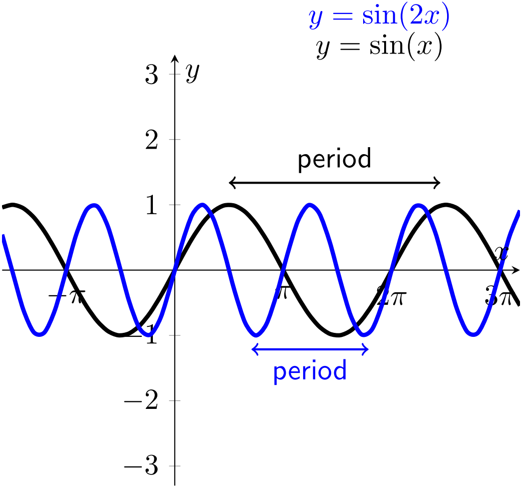

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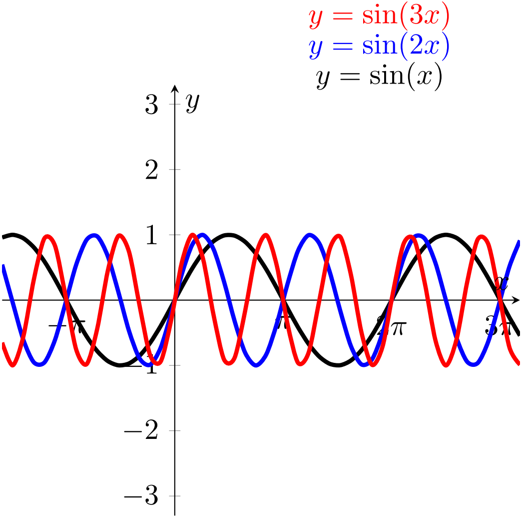

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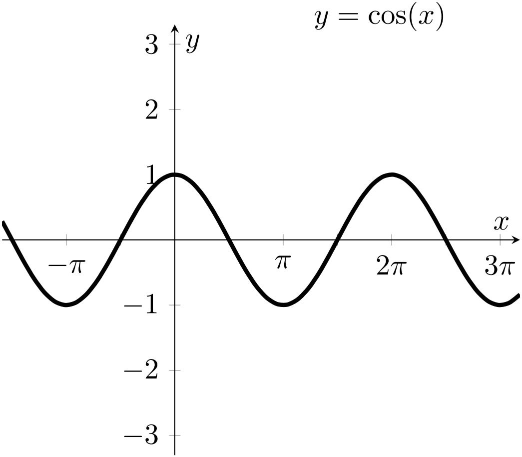

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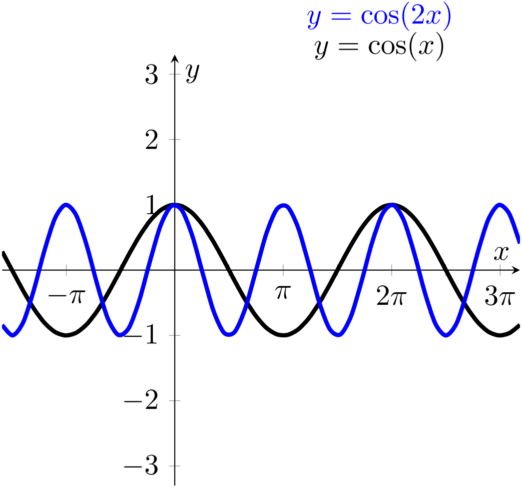

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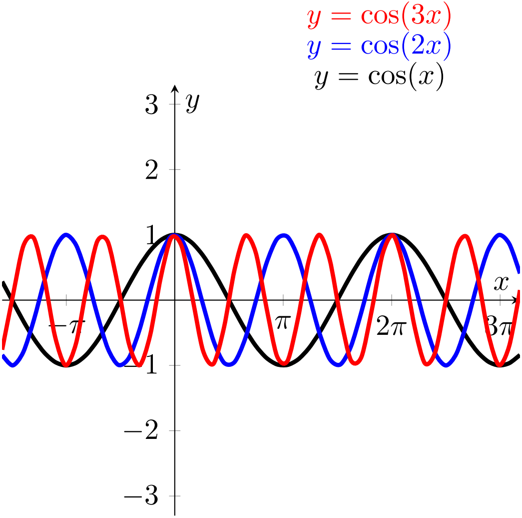
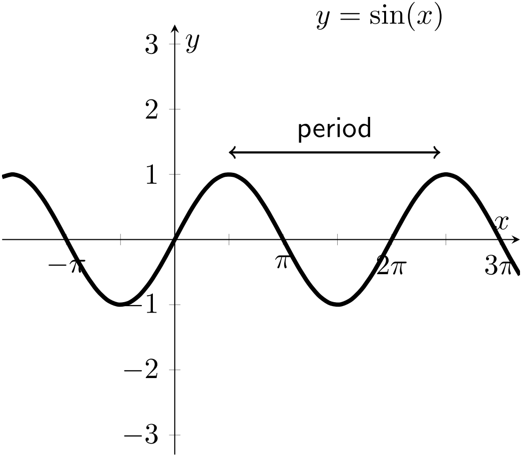


$y = \sin(x)$ vs. $y = \sin(\frac{1}{2}x)$



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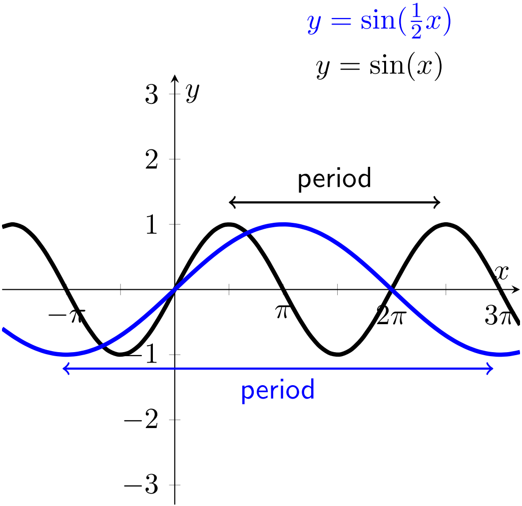

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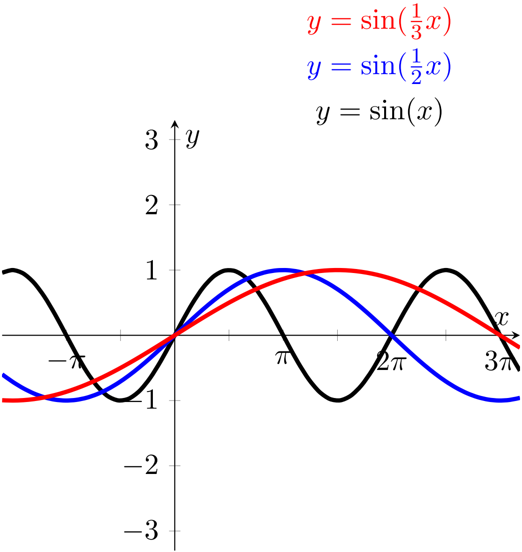

$y = \sin(bx)$ or $y = \cos(bx)$

Summary

The absolute value of b, namely $|b|$ $\leftrightsquigarrow$ inverse of horizontal stretching

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The absolute value of b, namely $|b| \leftrightsquigarrow$ inverse of horizontal stretching

$|b| > 1$

period decreases

$|b| < 1$

period increases

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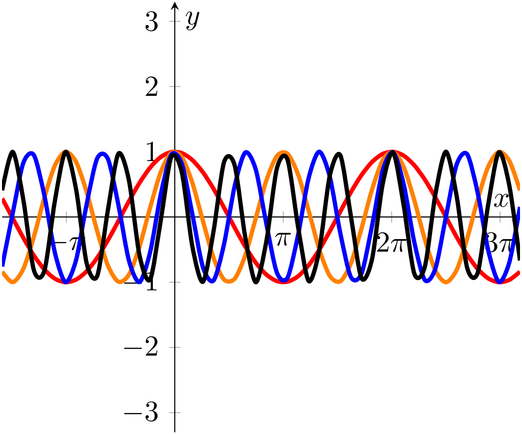
Definition

For $y = \sin(bx)$ or $y = \cos(bx)$, the number $\frac{2\pi}{|b|}$ is the **period**.

Summary with no words



$y = a\sin(bx)$ and $y = a\cos(bx)$

What happens when we combine a and b?

$y = a\sin(bx)$ and $y = a\cos(bx)$

What happens when we combine a and b?

For example, how do we graph $y = 3\sin(2x)$?

Theorem of Amplitudes and Periods

Let $a \neq 0$ and $b \neq 0$.

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Let $a \neq 0$ and $b \neq 0$.

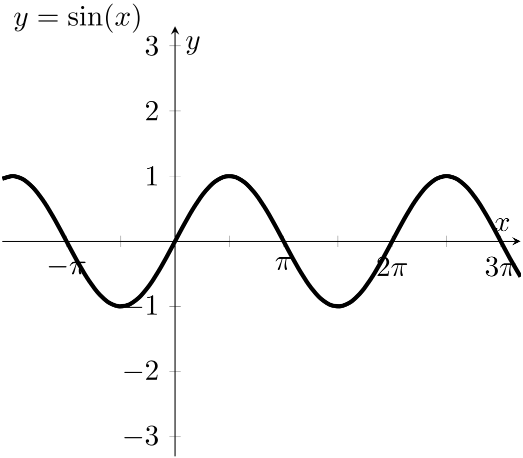
Theorem of Amplitudes and Periods

The graph of $y = a\sin(bx)$ or $y = a\cos(bx)$ has

- amplitude $|a|$
- period $\dfrac{2\pi}{|b|}$

Example: $y = 3\sin(2x)$

- amplitude $|a| = 3$
- period $\dfrac{2\pi}{|b|} = \dfrac{2\pi}{|2|} = \pi$



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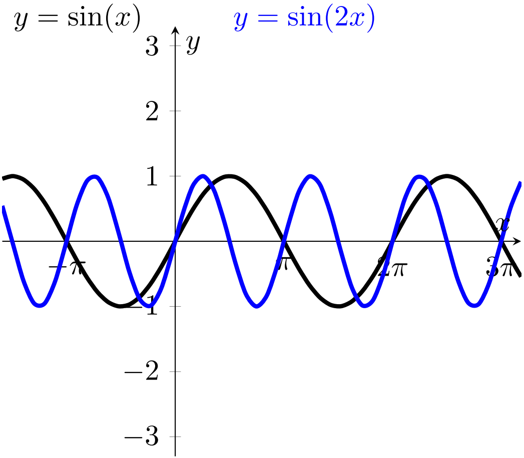

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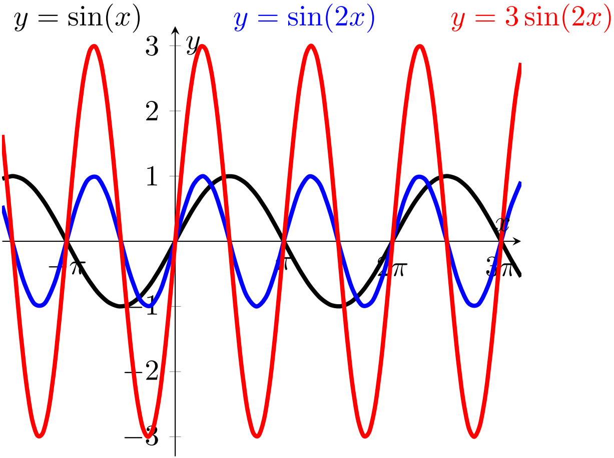

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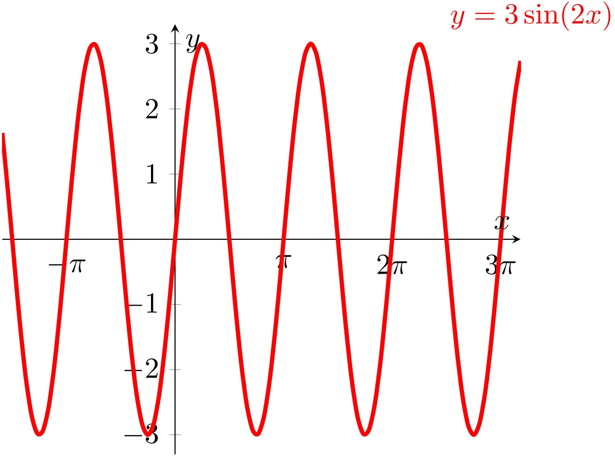

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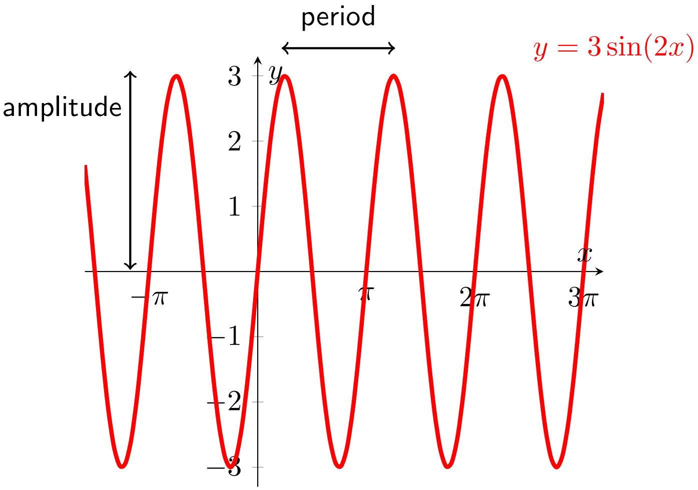
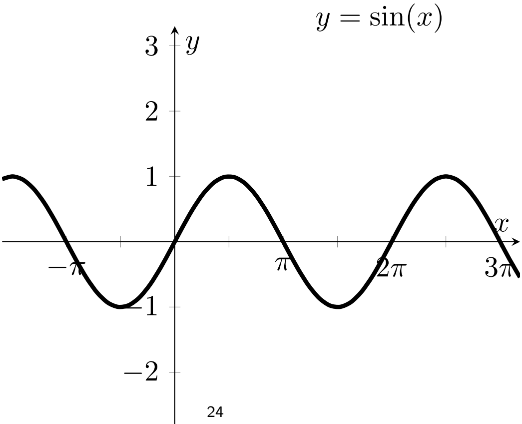


Example: $y = 2.5\sin(\frac{1}{2}x)$

- amplitude $|a| = 2.5$
- period $\dfrac{2\pi}{|b|} = \dfrac{2\pi}{|\frac{1}{2}|} = 4\pi$



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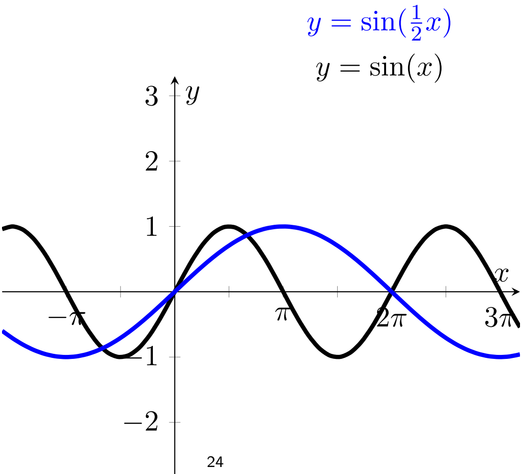

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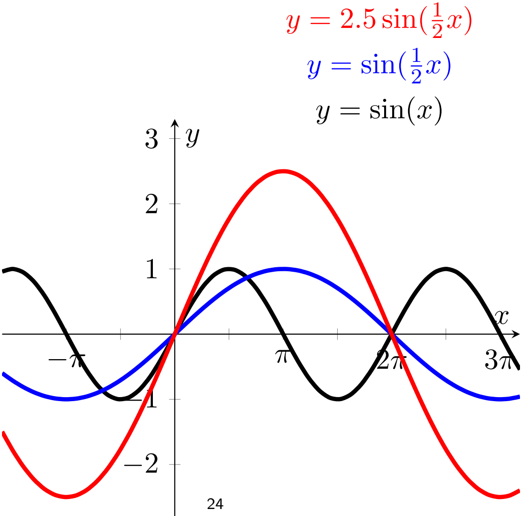

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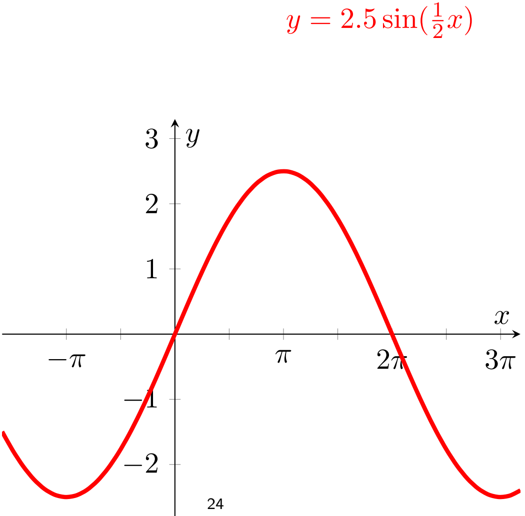

Example: $y = 2\cos(\pi x)$

- amplitude $|a| = 2$
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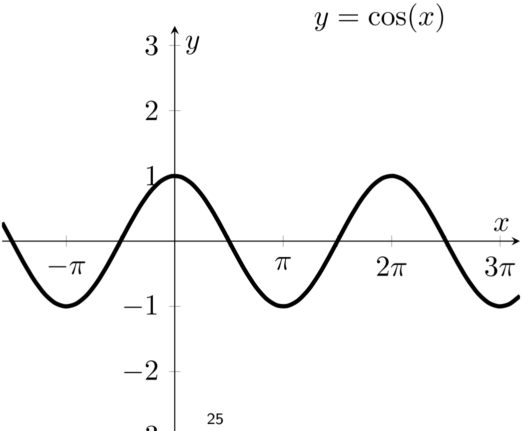

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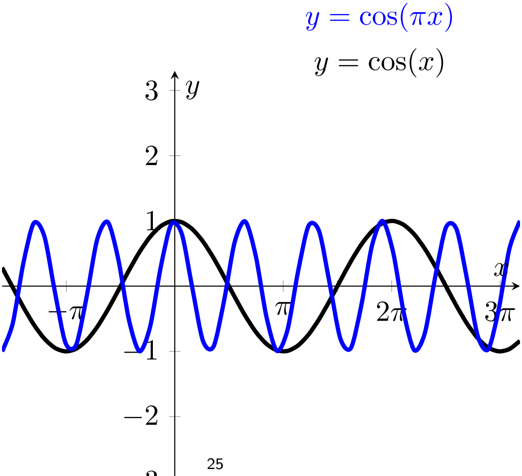

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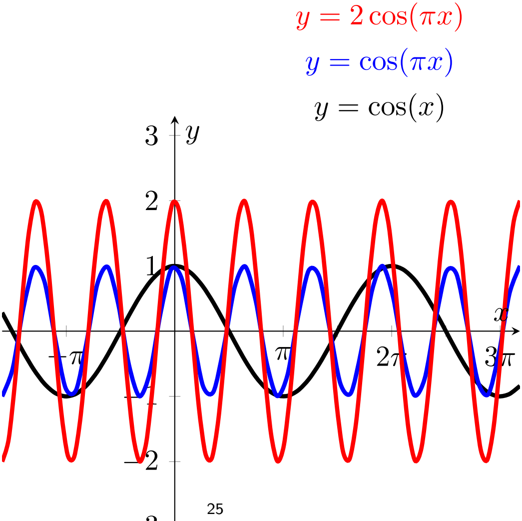

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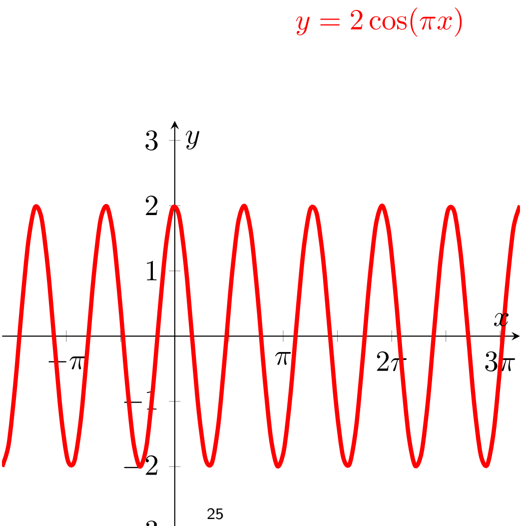

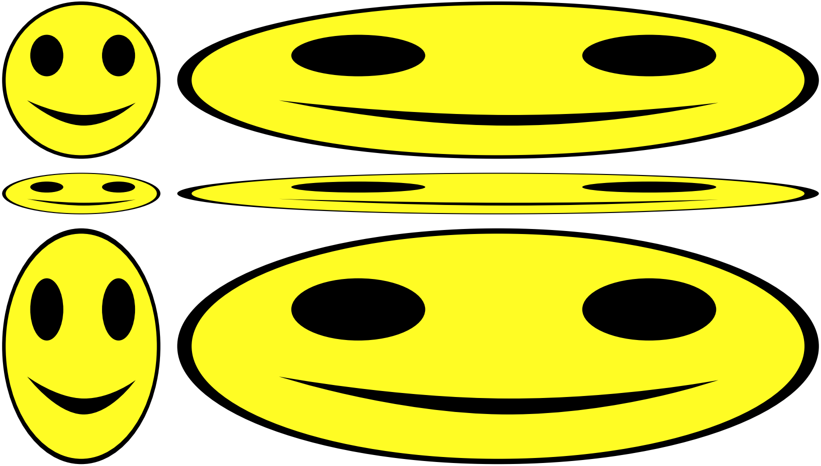

Last time...

We talked about how to graph

$$y = a\sin(bx) \quad \text{and} \quad y = a\cos(bx)$$

Key points:

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Key points:

- $a < 0$ "flips" the graph vertically
- $|a|$ is the amplitude. "vertical stretch"
  - vertical expansion if $|a| > 1$
  - vertical contraction if $|a| < 1$

Last time...

We talked about how to graph

$$y = a\sin(bx) \quad \text{and} \quad y = a\cos(bx)$$

Key points:

- $a < 0$ "flips" the graph vertically
- $|a|$ is the amplitude. "vertical stretch"
  - vertical expansion if $|a| > 1$
  - vertical contraction if $|a| < 1$
- $\frac{2\pi}{|b|}$ is the period
  - horizontal contraction if $|b| > 1$
  - horizontal expansion if $|b| < 1$

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What happens when we combine b and c?

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For example, how do we graph $y = \sin(2x + \frac{\pi}{2})$?

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The number $-\dfrac{c}{b}$ is called the **phase shift**.

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The number $-\dfrac{c}{b}$ is called the **phase shift**.

- phase shift negative $\leftrightsquigarrow$ horizontal shift to the left
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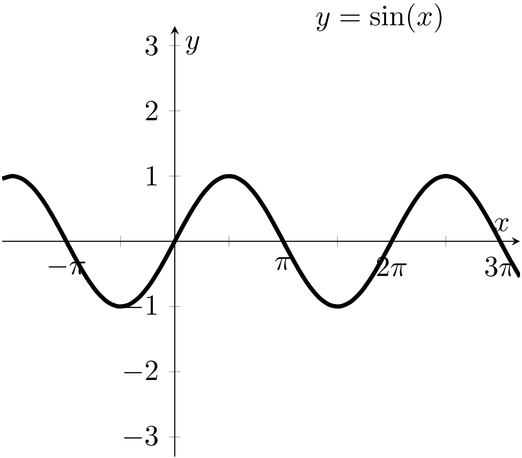

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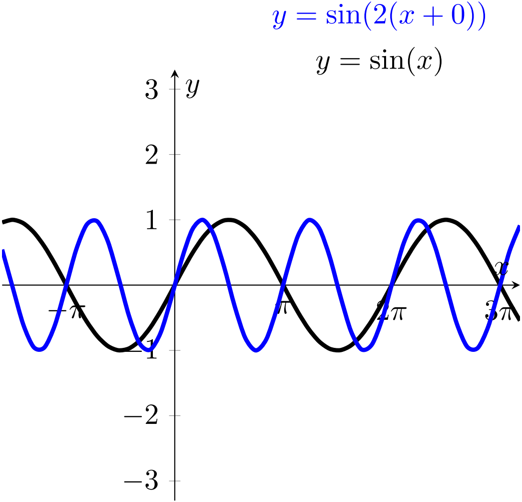

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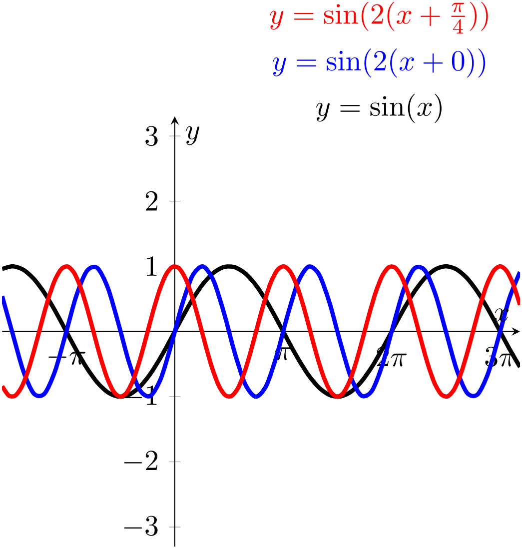

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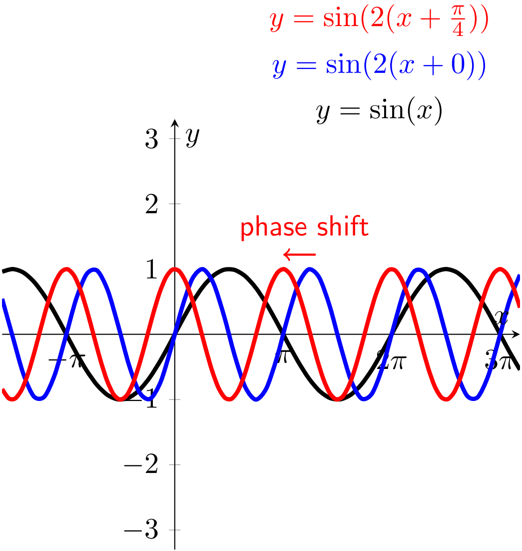

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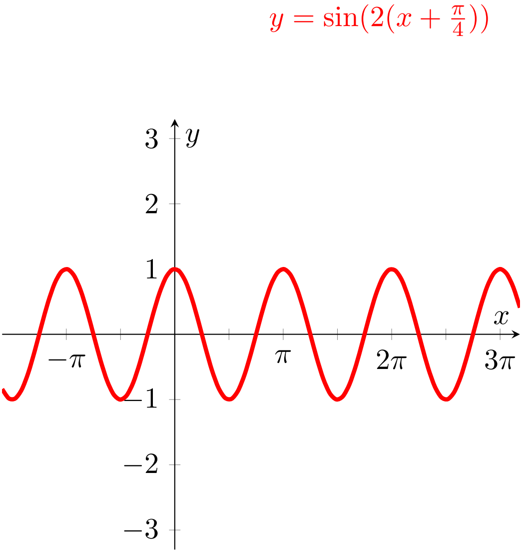

Where does a cycle begin/end?

Consider

$$y = \sin(\underbrace{bx + c}_{\text{input to sine}})$$

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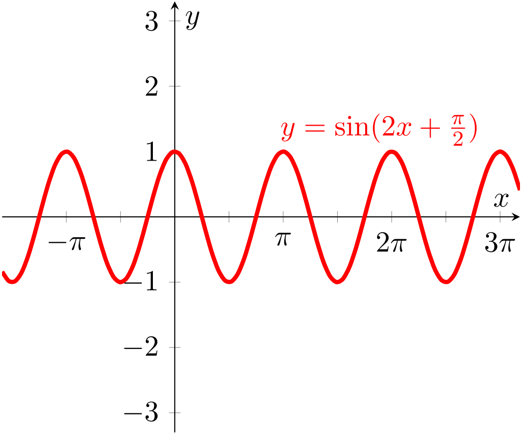
Example: in pictures!!



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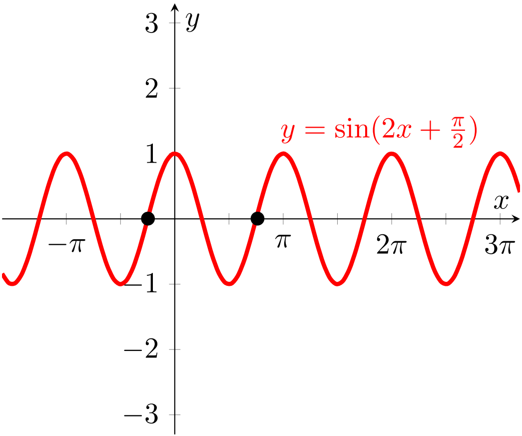

Graph $y = \cos(3x - \pi)$

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Rewrite

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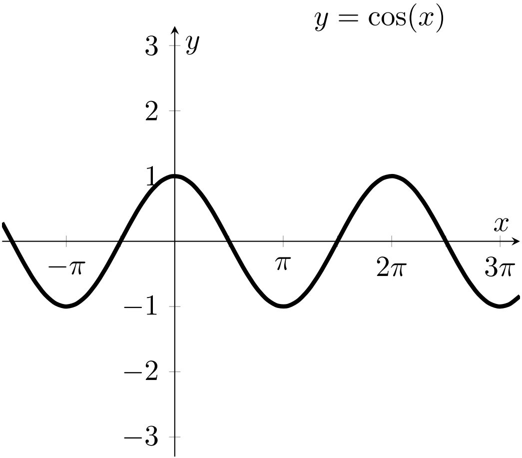
A "cosine" cycle... $(y = 1)$

- starts at $x = \frac{\pi}{3}$
- ends at $x = \pi$

Graphing $y = \cos\left[3\left(x - \frac{\pi}{3}\right)\right]$



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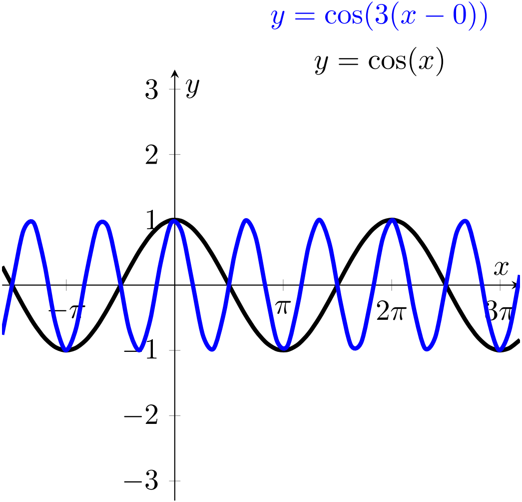

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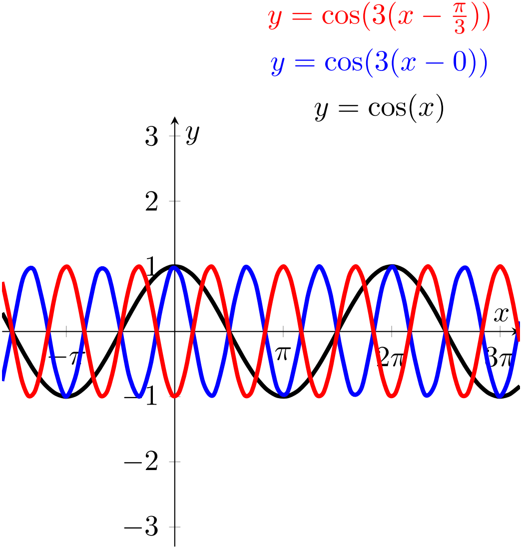

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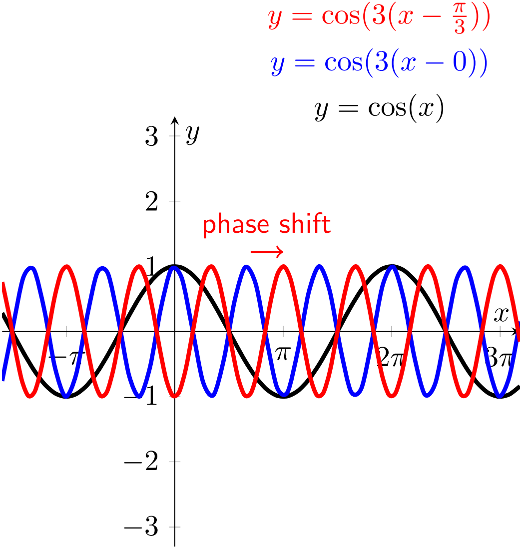

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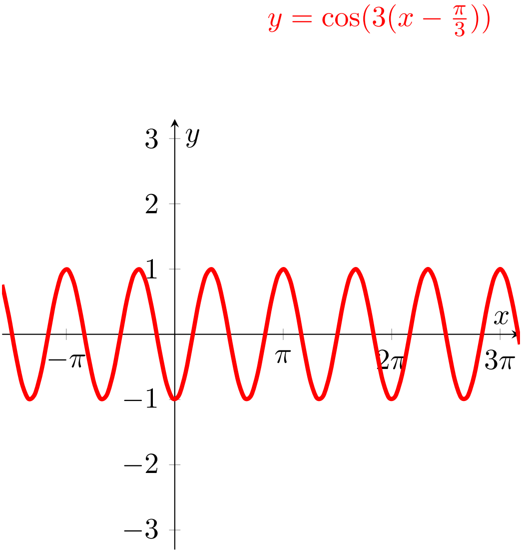

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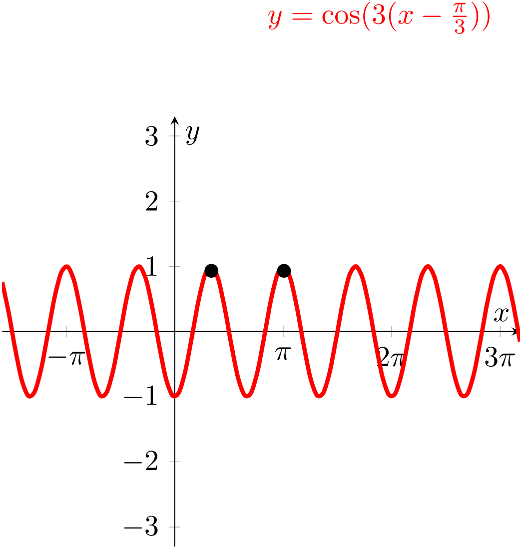
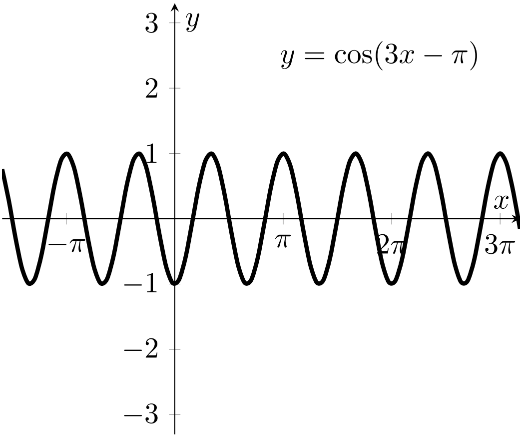


a, b, c

Graph a function of the form $y = a\sin(bx + c)$

Example: Graph $y = 2\cos(3x - \pi)$



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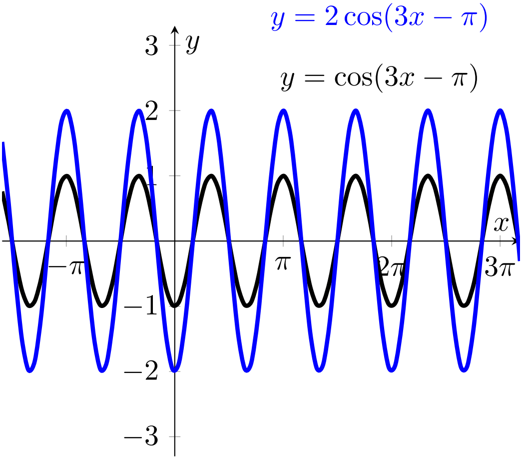
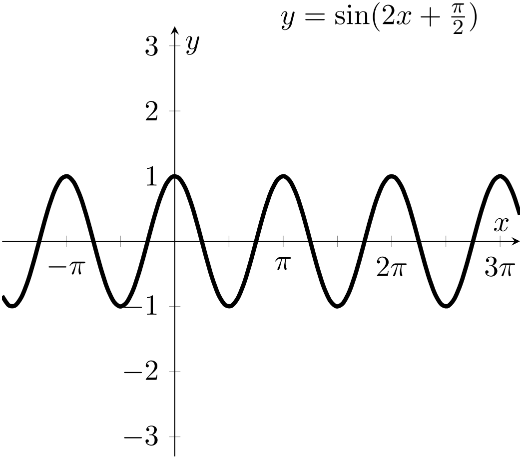


Strategy for graphing $y = a\sin(bx + c)$

1. Ignore a first: Graph $y = \sin(bx + c)$ like before (period, phase shift)
2. “Stretch” by a vertically

Example: Graph $y = 3\sin(2x + \frac{\pi}{2})$



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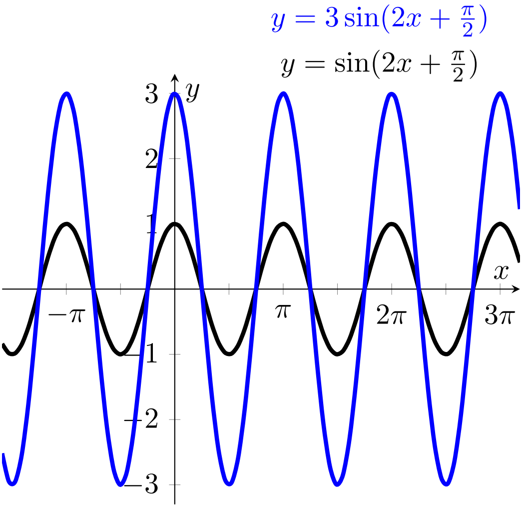

Putting it all together:
Graphing $y = a\sin(bx + c) + d$
or $y = a\cos(bx + c) + d$

So, what does the d do?

Compare the graphs of $y = a\sin(bx + c) + d$ or $y = a\cos(bx + c) + d$

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The number d is called the **vertical shift**.

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- $d < 0 \leftrightsquigarrow$ shift downwards

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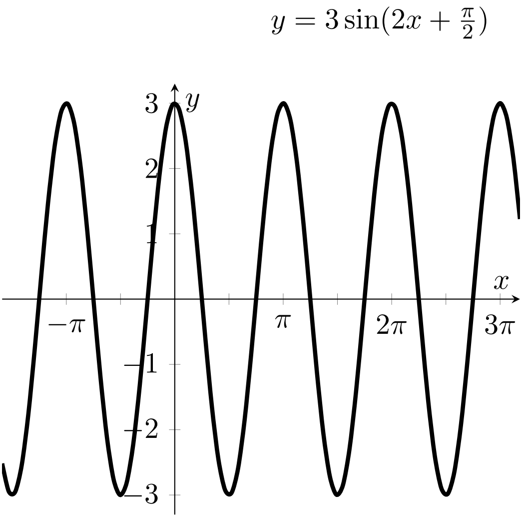
Strategy

Take care of d last!

Example: Graph $y = 3\sin(2x + \frac{\pi}{2}) + 1$



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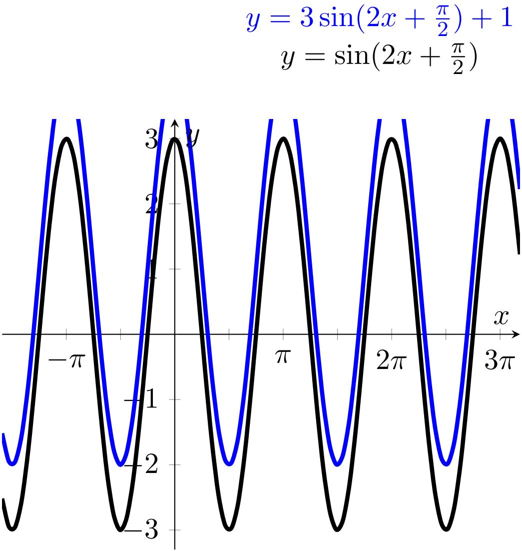

Example: Graph $y = \frac{1}{2}\sin(2x + \frac{\pi}{2}) - \frac{1}{6}$

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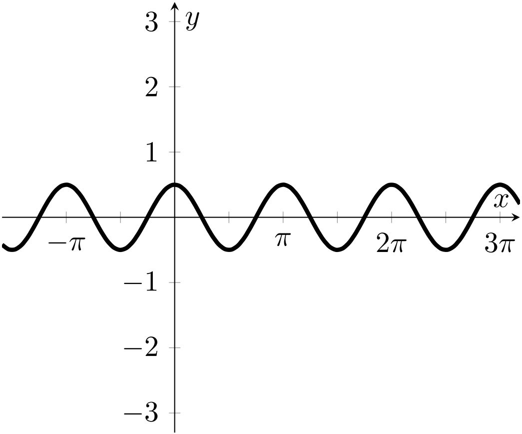
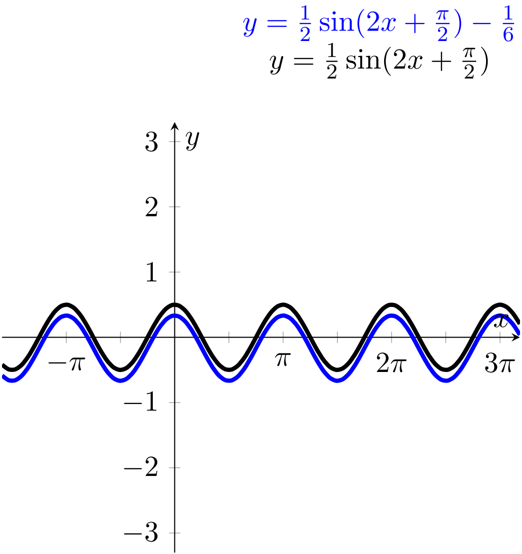


Example: Graph $y = \frac{1}{2}\sin(2x + \frac{\pi}{2}) - \frac{1}{6}$

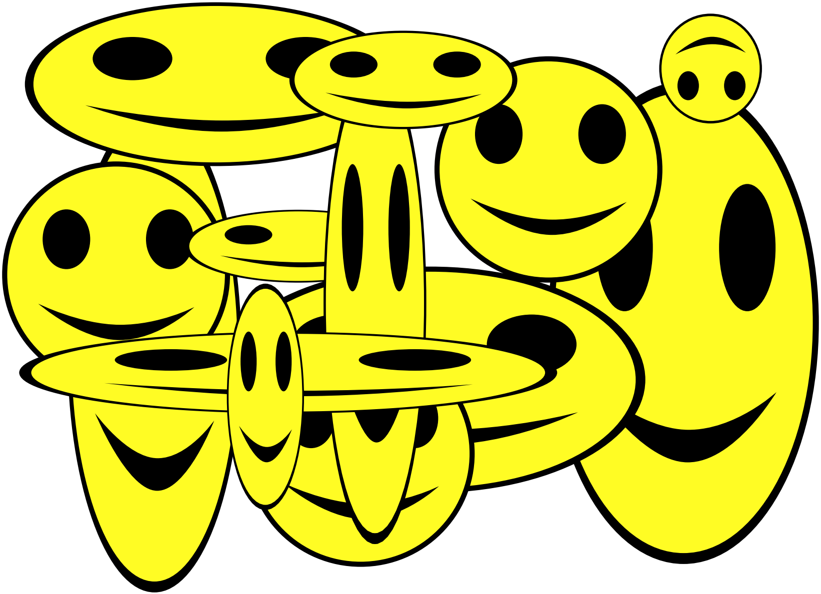


Graph transformations of $\sin$ and $\cos$

A summary

Given $y = a\sin(bx + c) + d$ or $y = a\cos(bx + c) + d$

1. Process b and c first: get the period and phase shift
2. Stretch vertically using a
3. Shift vertically using d



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