# 6.6: More Trigonometric Graphs

E. Kim

MTH 151

All notation and terminology is based on Swokowski, Cole. *Algebra and Trigonometry: with analytic geometry*. Classic 12th Edition.

# Goal

Last time we graphed

$$y = a\sin(bx + c)$$

$$y = a\cos(bx + c)$$

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Last time we graphed

$$y = a\sin(bx + c)$$

$$y = a\cos(bx + c)$$

Build the graphs of

$$y = a\csc(bx + c)$$

$$y = a\sec(bx + c)$$

$$y = a\tan(bx + c)$$

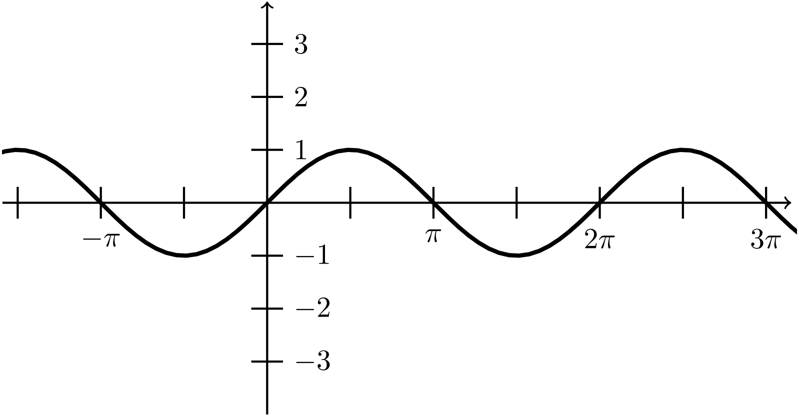
$$y = a\cot(bx + c)$$

step-by-step

# Review of Section 6.3

# $y = \sin x$ and $y = \csc x$

Reciprocal identity $\csc x = \frac{1}{\sin x}$

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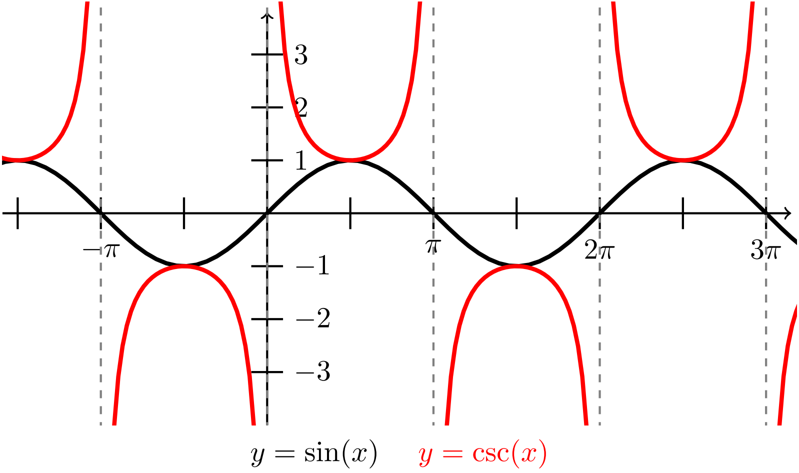

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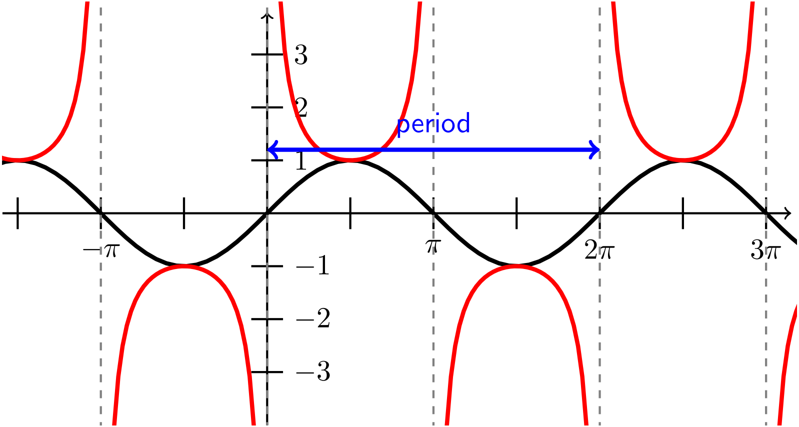
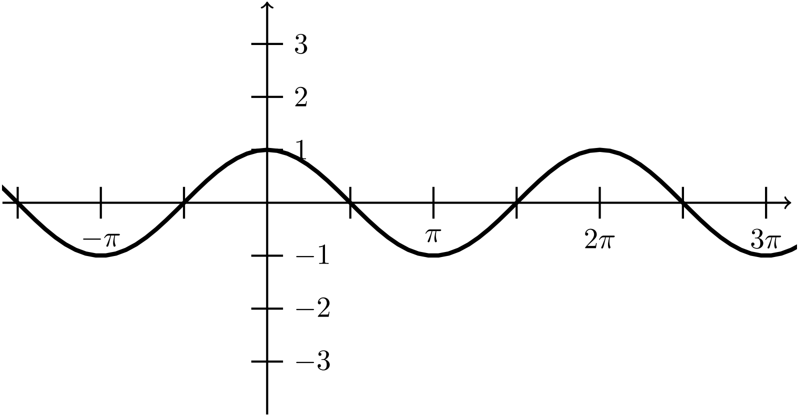



$y = \sin(x)$ $y = \csc(x)$

both functions have period $2\pi$

# $y = \cos x$ and $y = \sec x$

Reciprocal identity $\sec x = \frac{1}{\cos x}$

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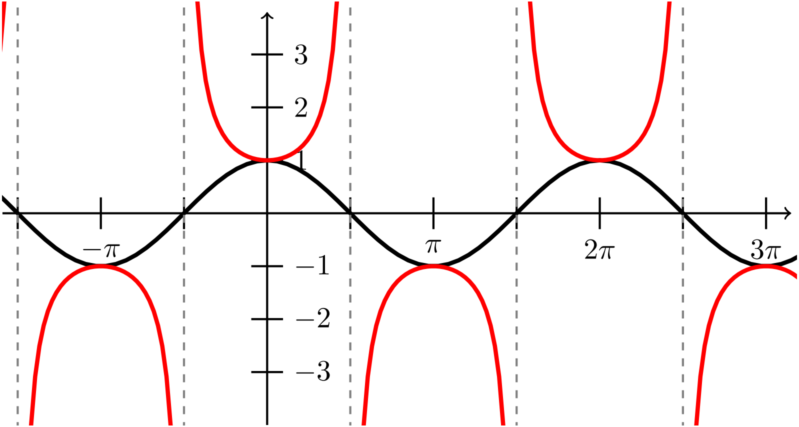


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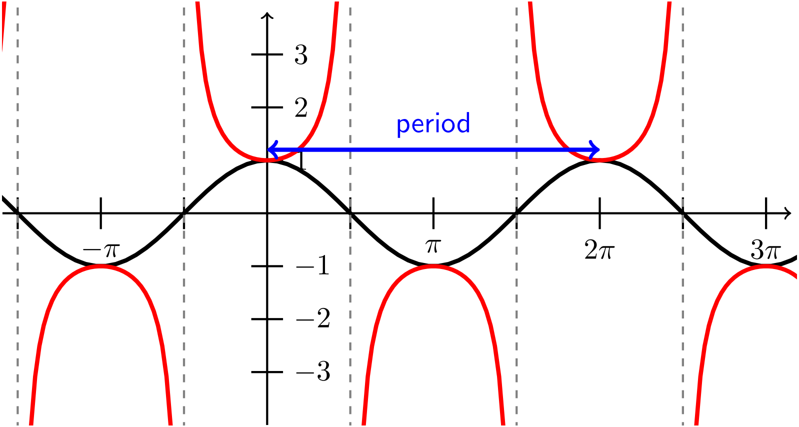


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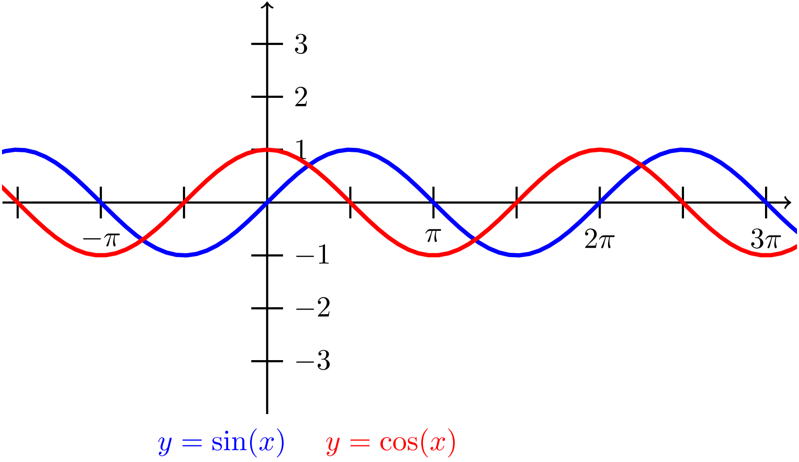

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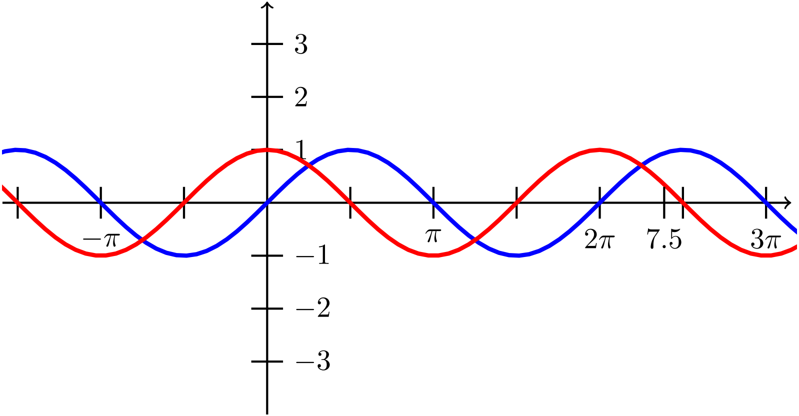


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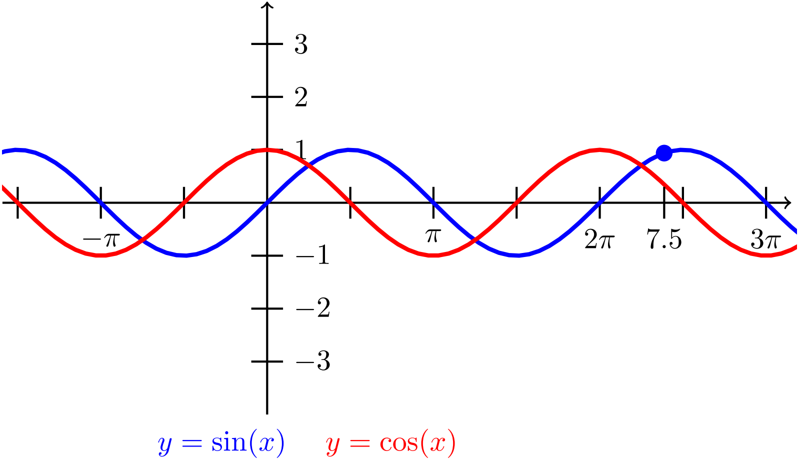

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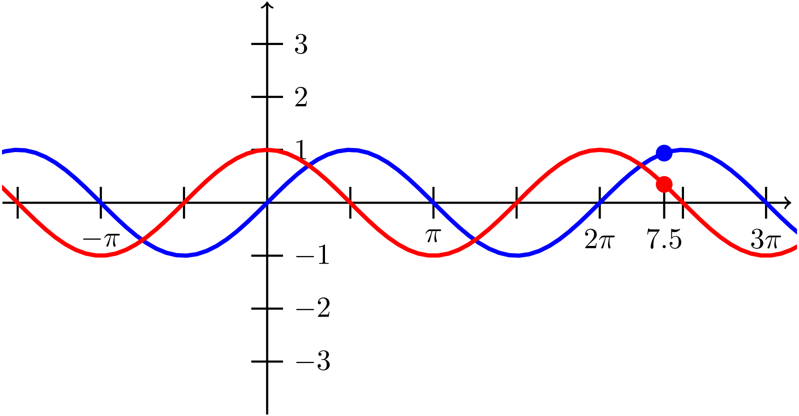


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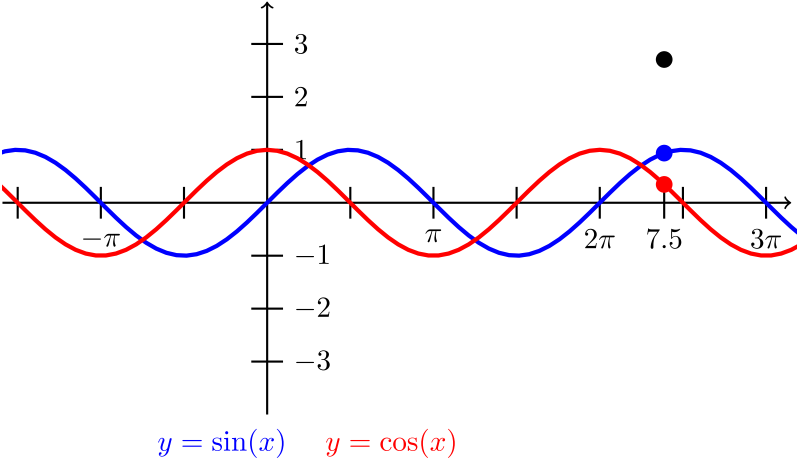

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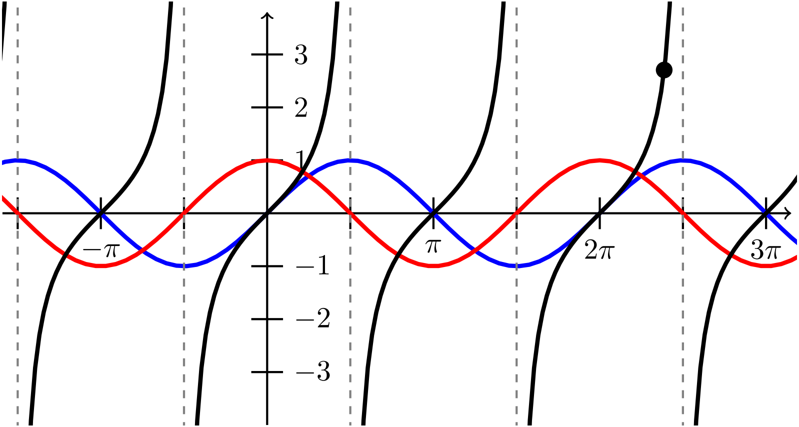


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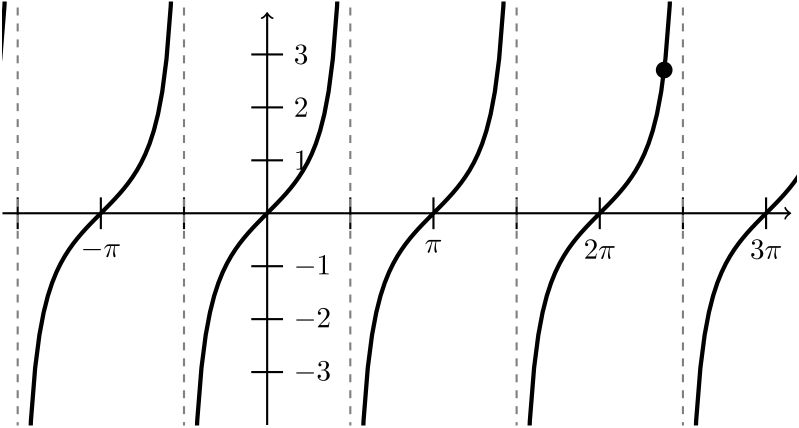


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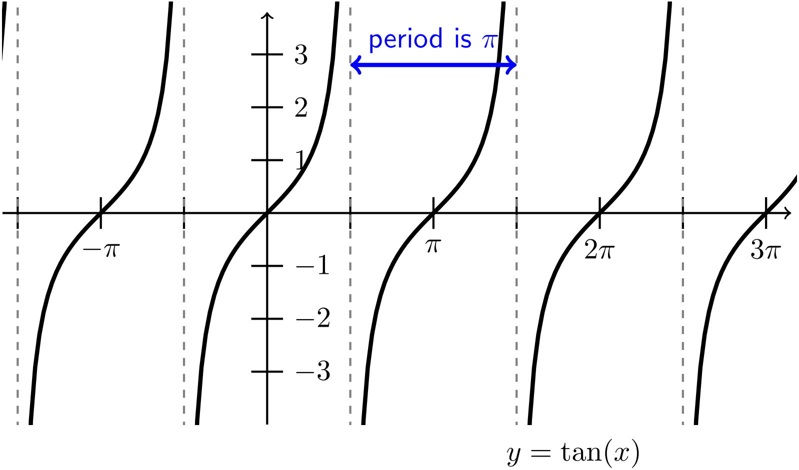
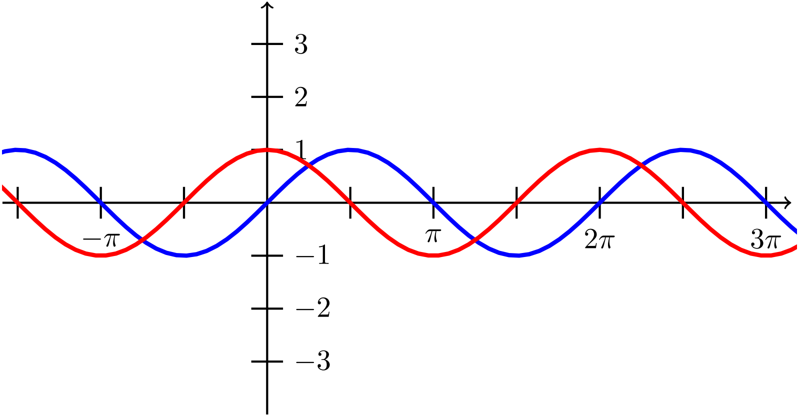



period of $\tan x$ is $\pi$

# Use $y = \sin x$ and $y = \cos x$ to get $y = \cot x$

$\cot x = \frac{\cos x}{\sin x}$

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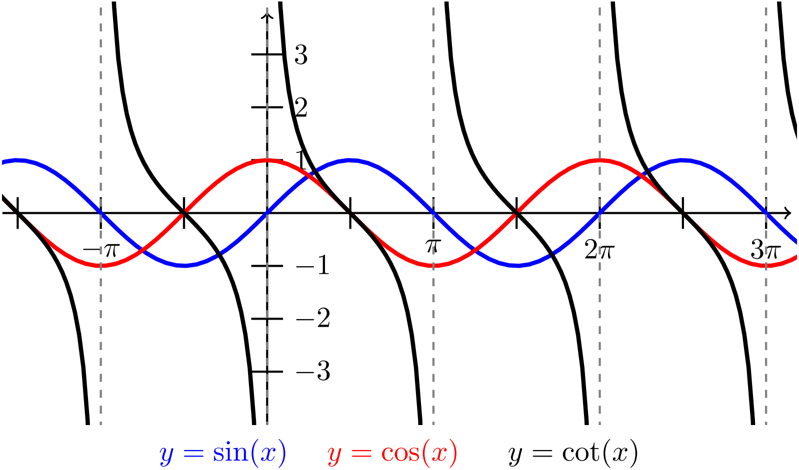

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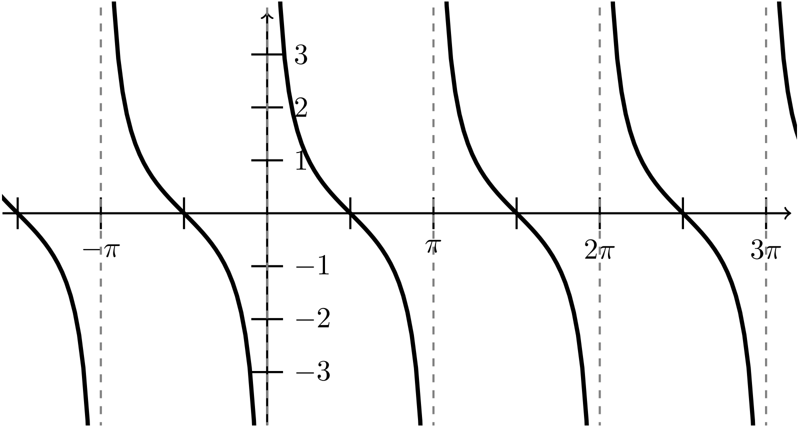


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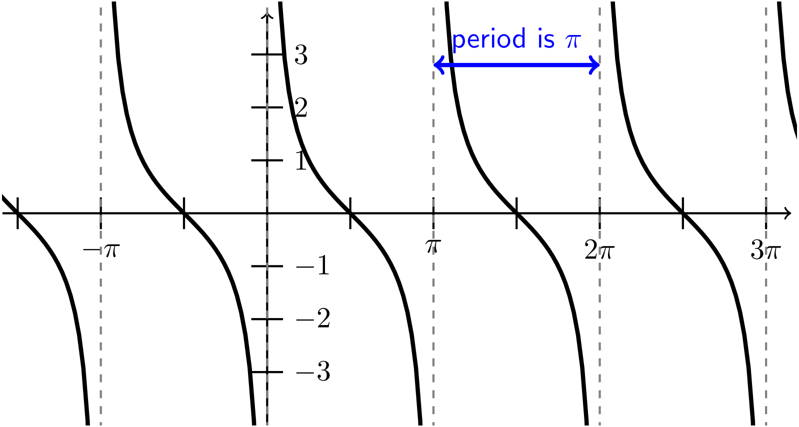


$y = \cot(x)$

period of $\cot x$ is $\pi$

# In the graphs of $y = \csc x$, $y = \sec x$, $y = \tan x$, $y = \cot x$

There is no "biggest" or "smallest" $y$ value occuring:

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No notion of amplitude

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There is no “biggest” or “smallest” $y$ value occuring:

No notion of amplitude (contrast: $y = \sin x$ or $y = \cos x$)

# Goal

Build the graphs of

$$y = a\csc(bx + c)$$

$$y = a\sec(bx + c)$$

$$y = a\tan(bx + c)$$

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step-by-step

# How about just $a$?

$$y = a \csc x$$

$$y = a \sec x$$

$$y = a \tan x$$

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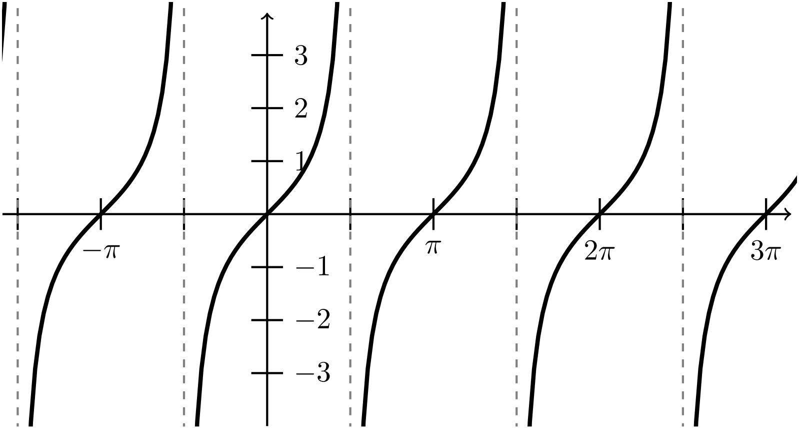
# $y = \tan x$ vs $y = 2\tan x$ vs $y = 3\tan x$

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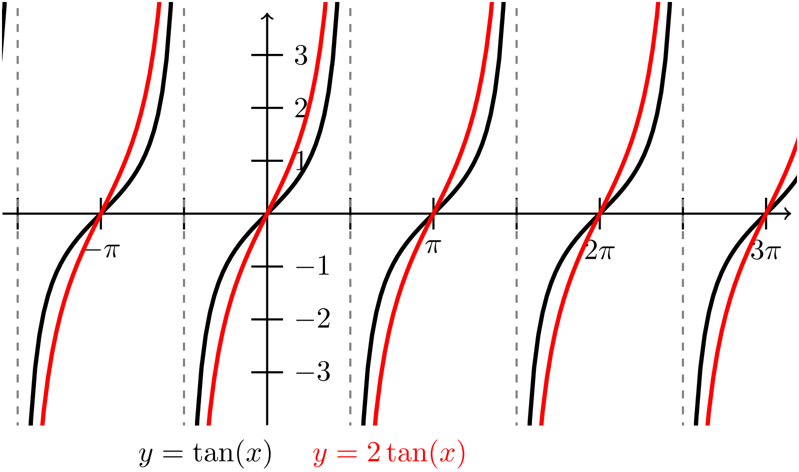

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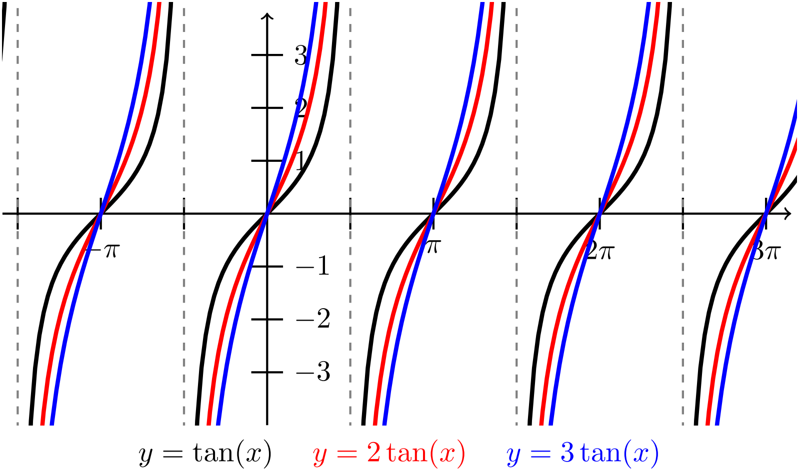

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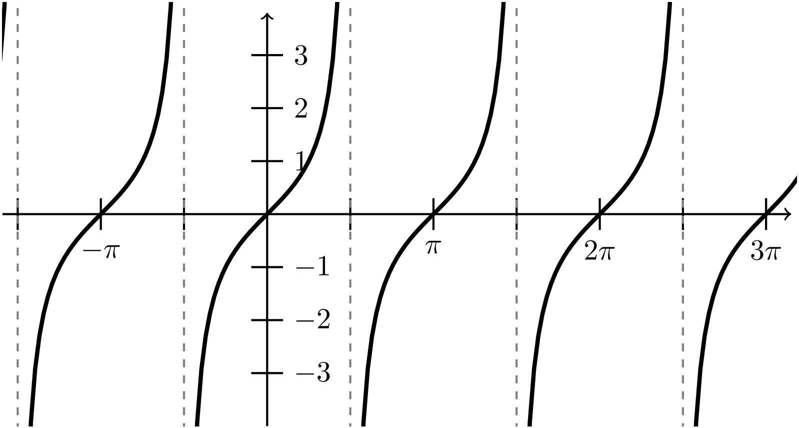


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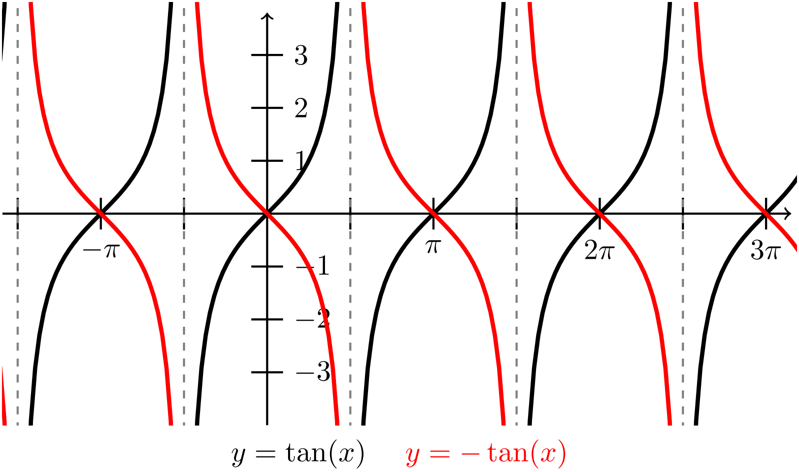
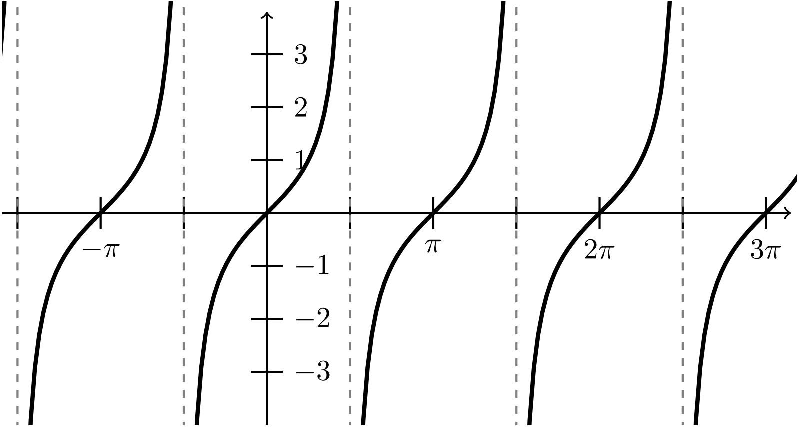


## $y = \tan x$ vs $y = \frac{1}{2} \tan x$ vs $y = \frac{1}{3} \tan x$

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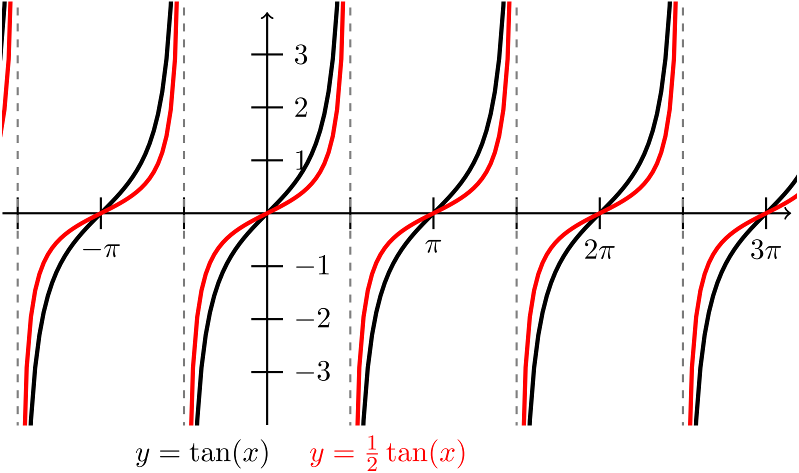

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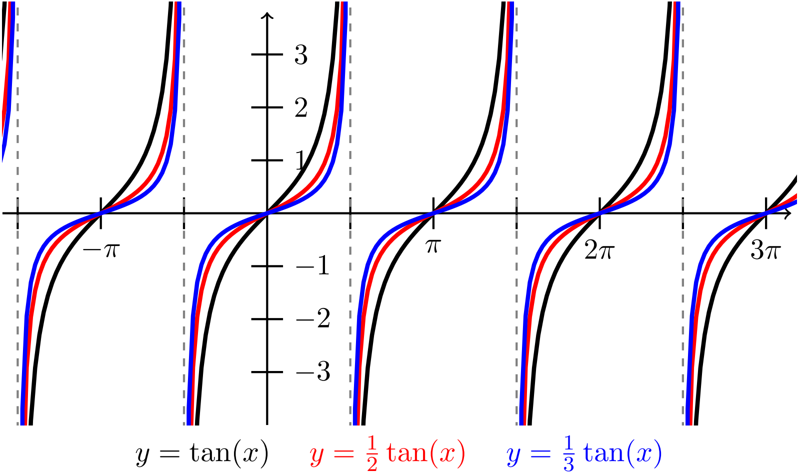

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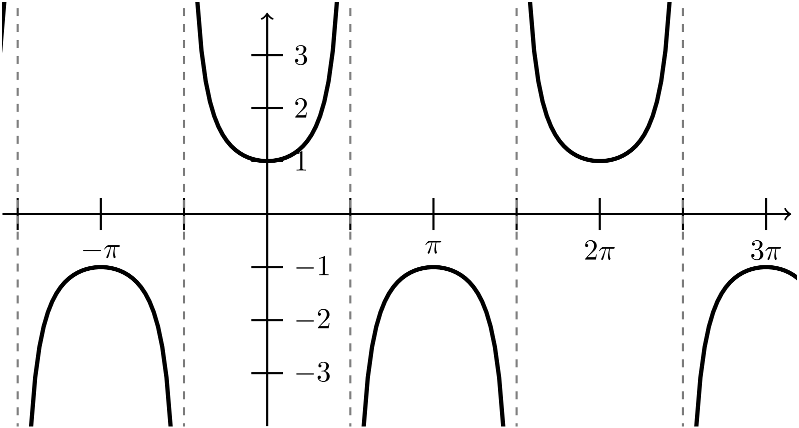


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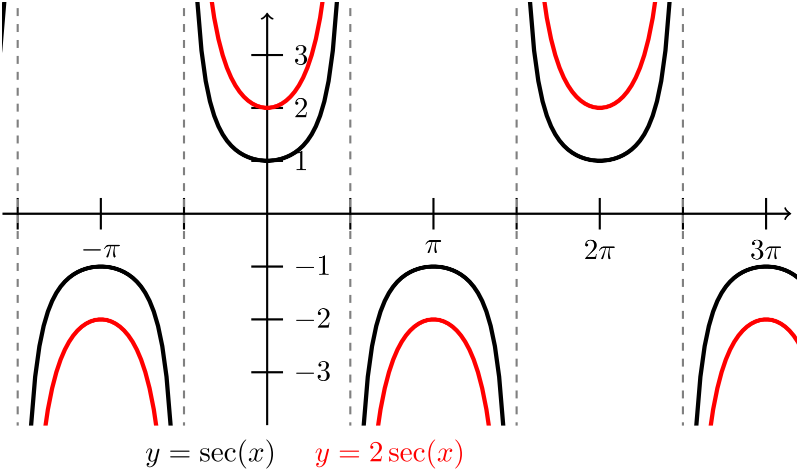

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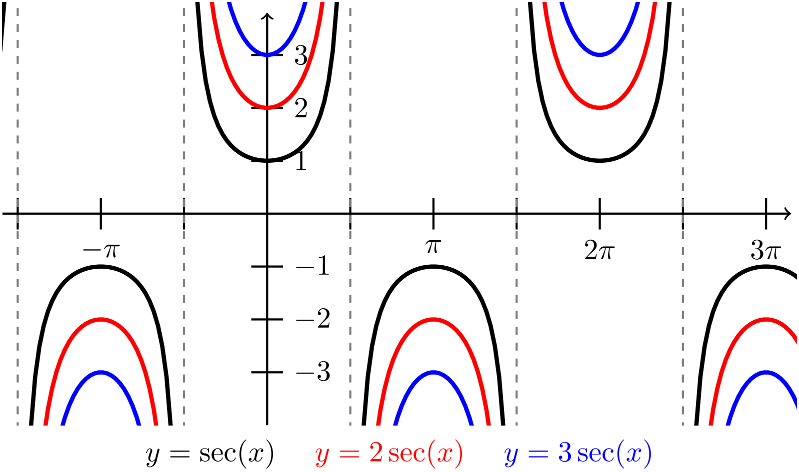

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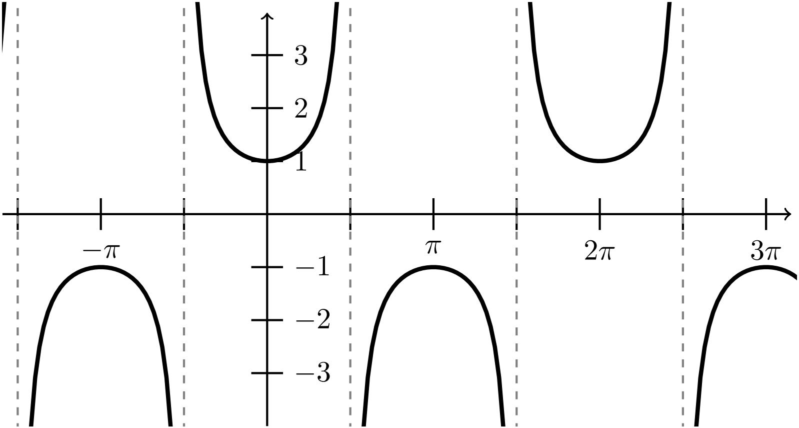


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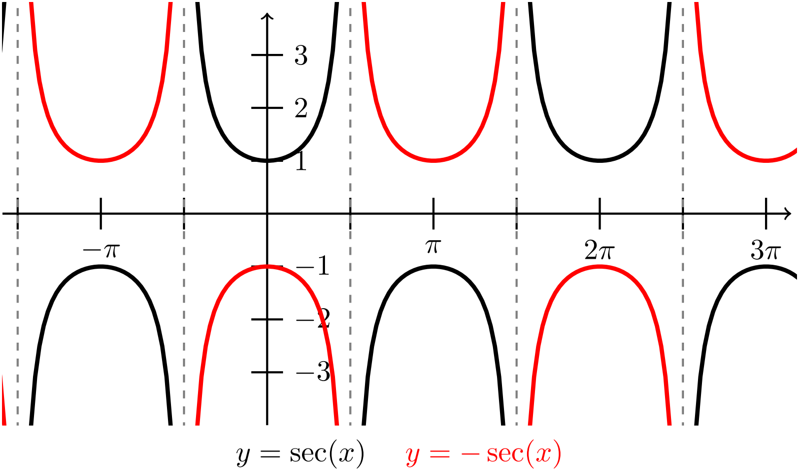

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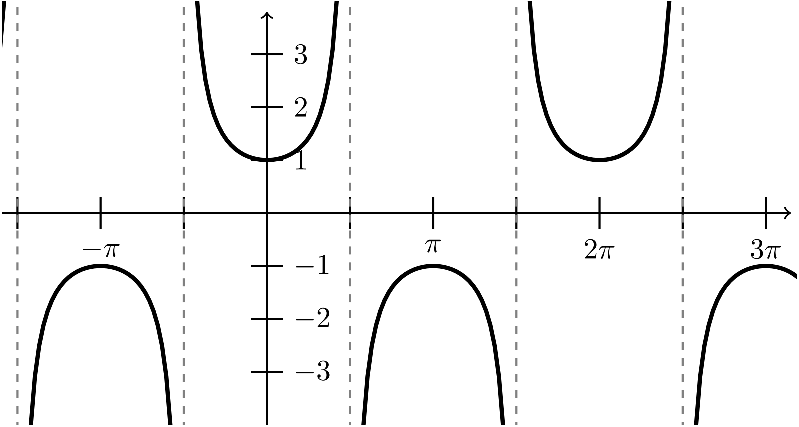

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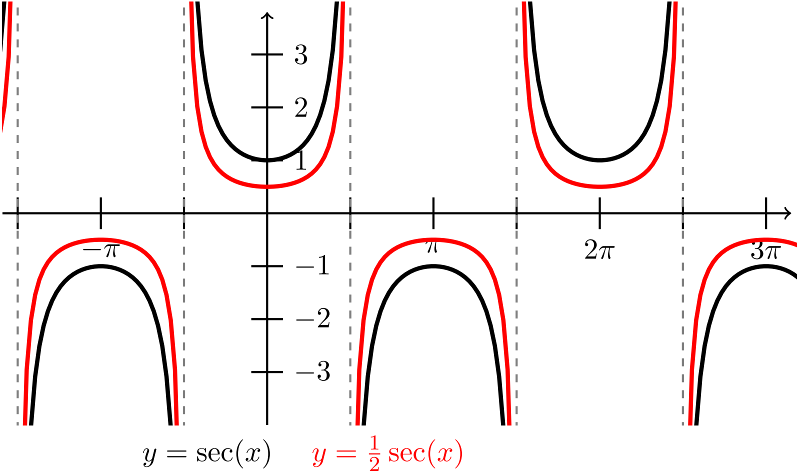

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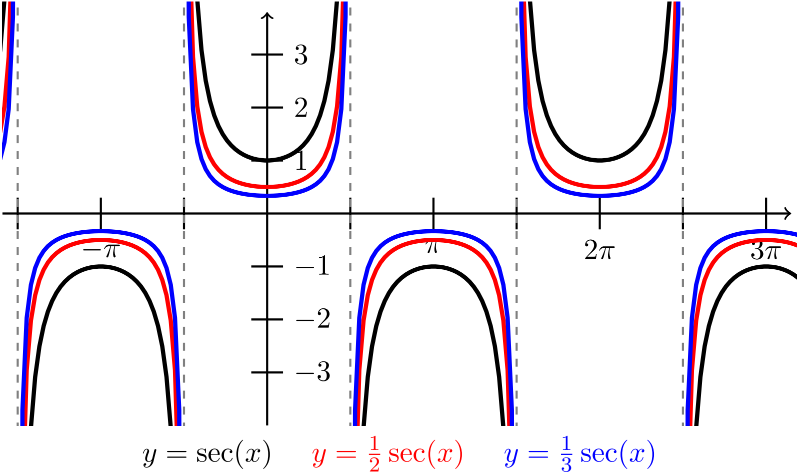

# What does $a$ do?

Sign of $a$:

- $a > 0 \leftrightsquigarrow$ don't flip
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Size of $|a|$:

- $|a| > 1 \leftrightsquigarrow$ vertical expand
- $|a| < 1 \leftrightsquigarrow$ vertical contract

# How about just $b$?

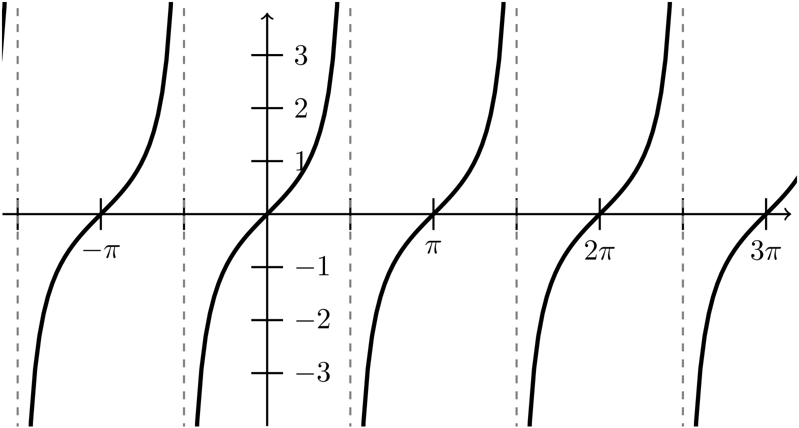
$$y = \csc bx$$

$$y = \sec bx$$

$$y = \tan bx$$

$$y = \cot bx$$

# $y = \tan bx$

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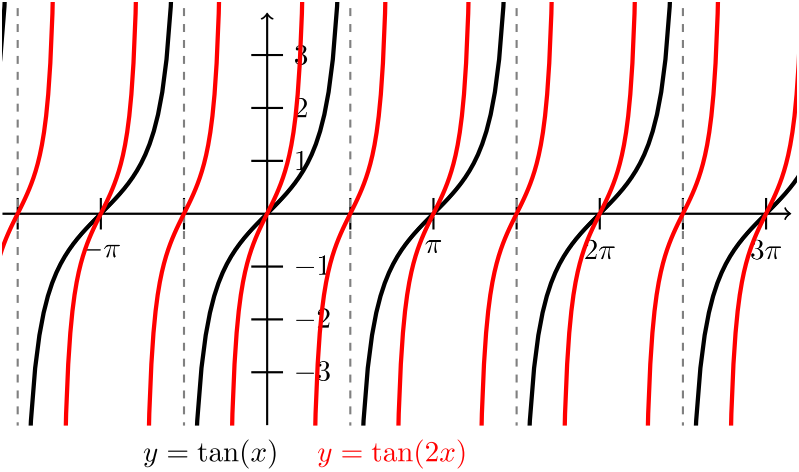

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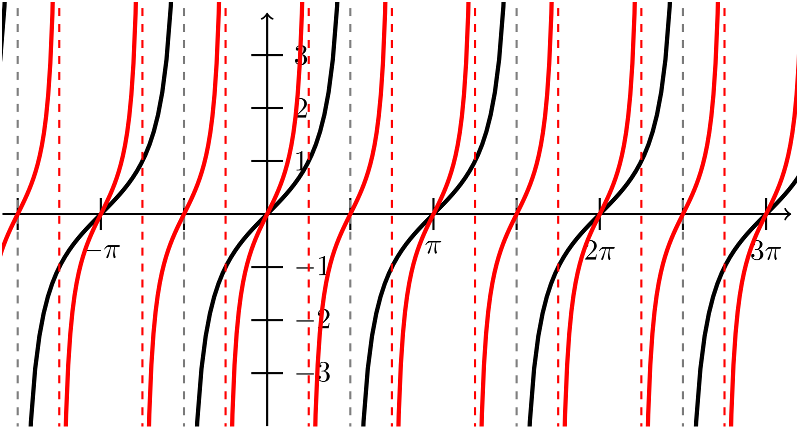


$y = \tan(x)$ $\quad$ $y = \tan(2x)$

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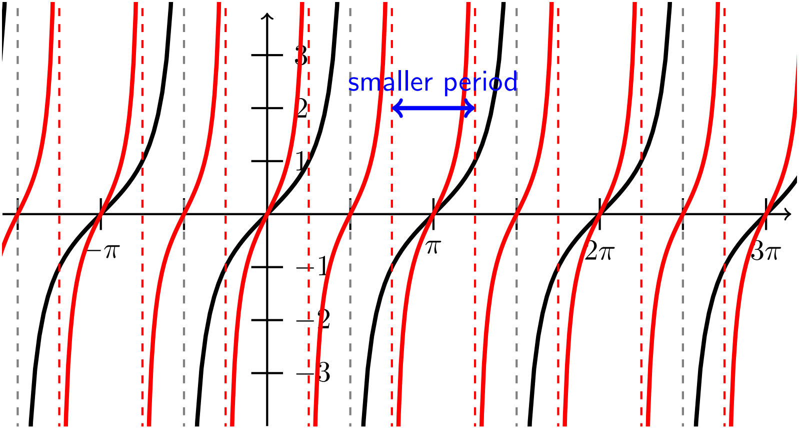


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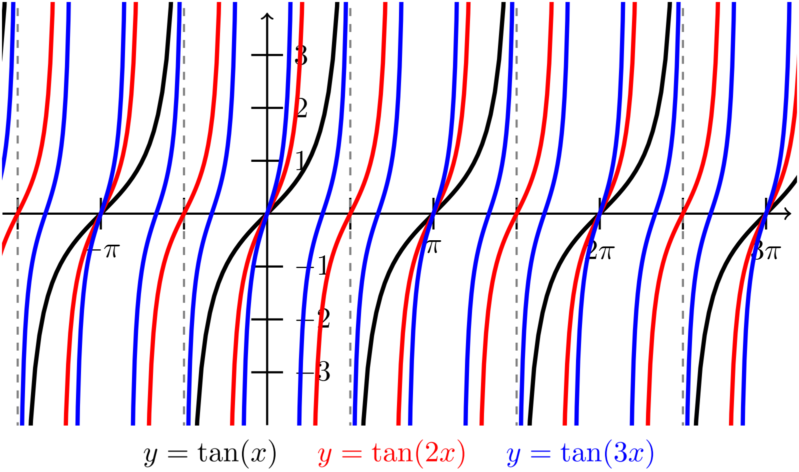
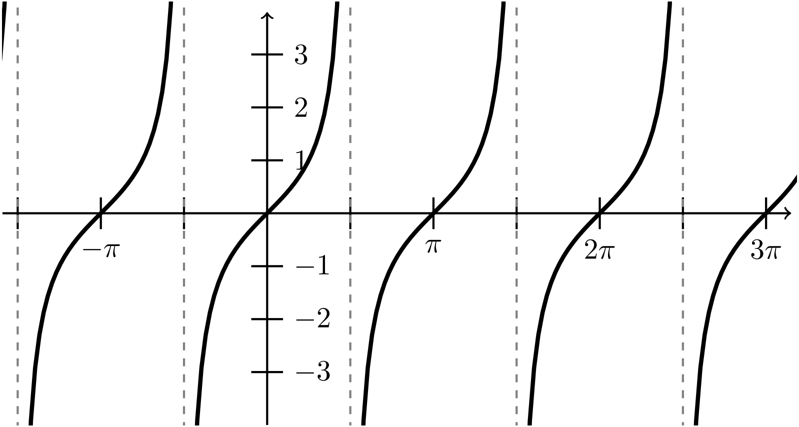


## $y = \tan bx$ where $b$ is a fraction

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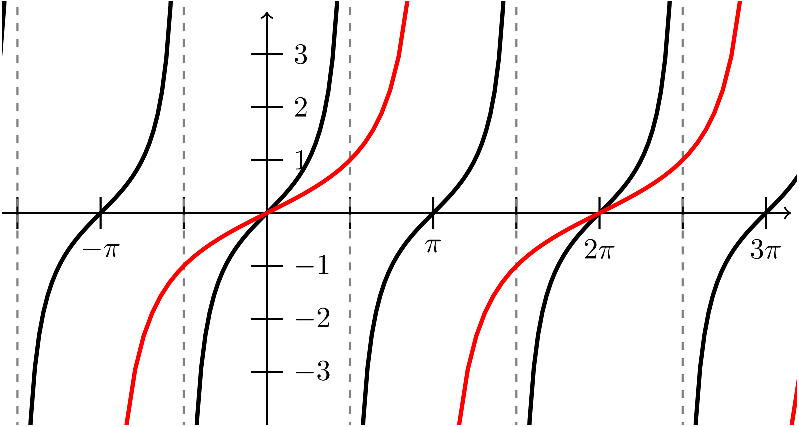
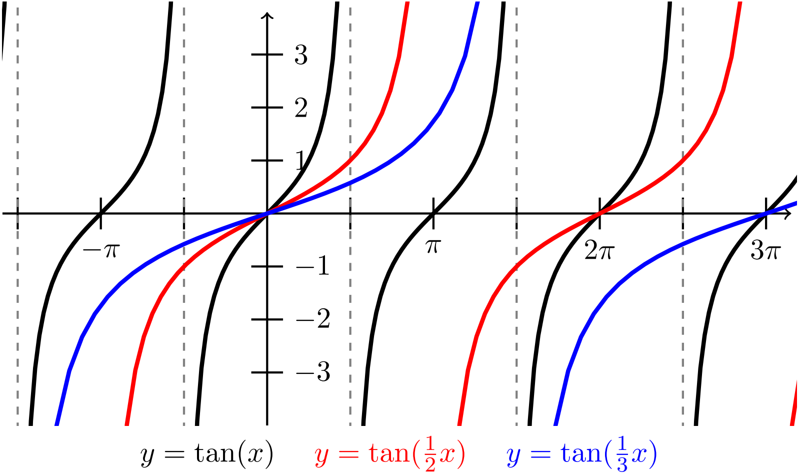



$y = \tan(x)$ $\quad$ $y = \tan(\frac{1}{2}x)$

# $y = \tan bx$ where $b$ is a fraction

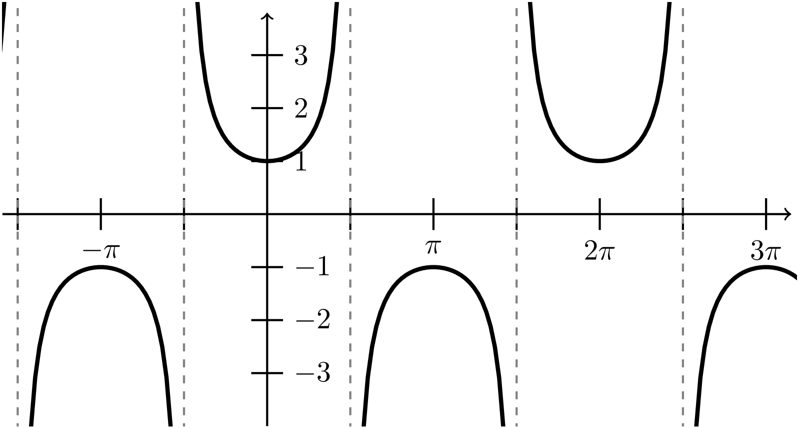


# $b$ in $y = \tan(bx)$ or $y = \cot(bx)$

## What does $b$ do?

- Since original period of tangent and cotangent is $\pi$
- For $y = \tan(bx)$ or $y = \cot(bx)$, period is $\frac{\pi}{|b|}$

$y = \sec bx$

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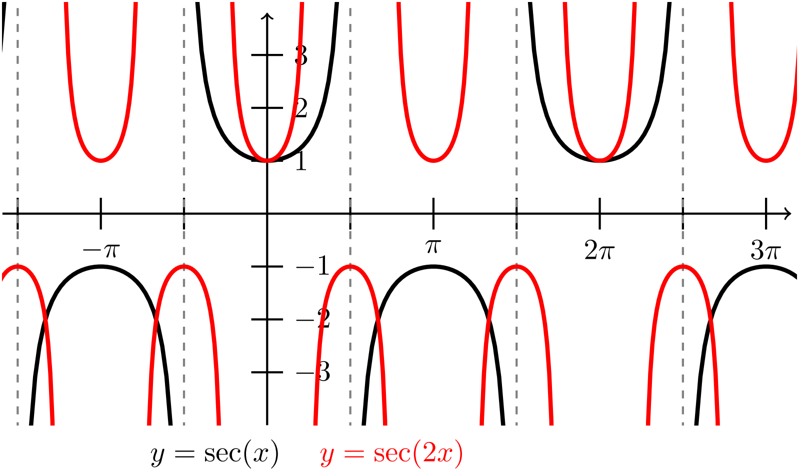

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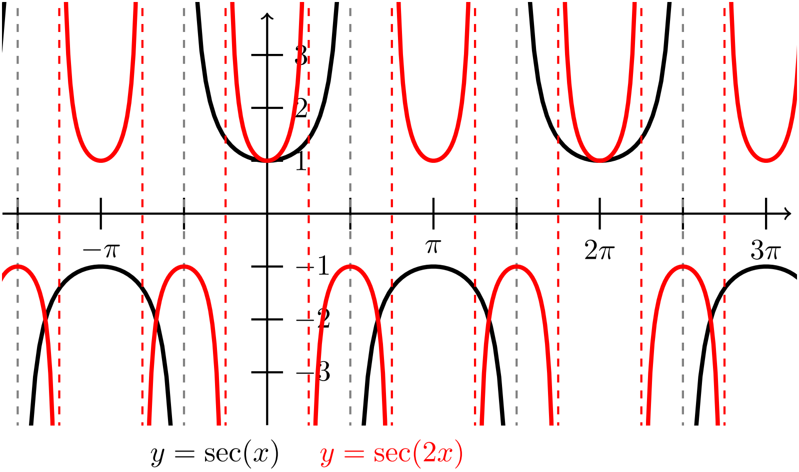

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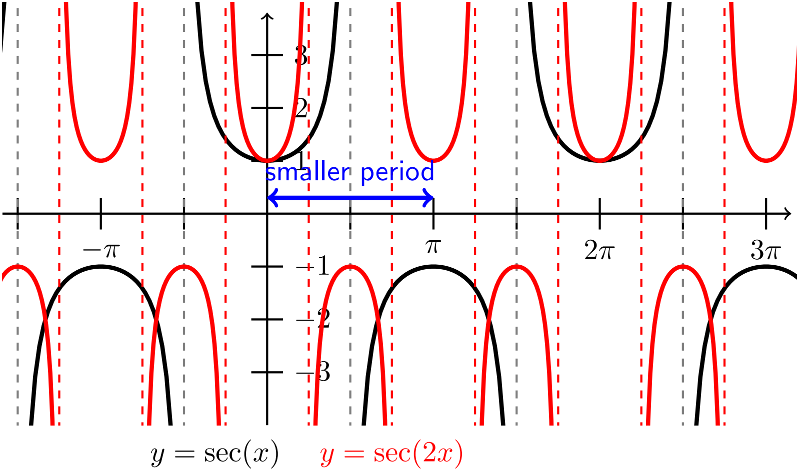

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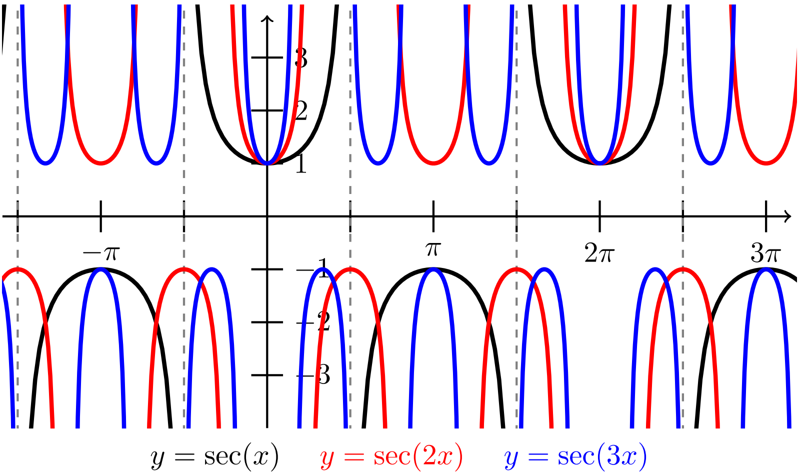

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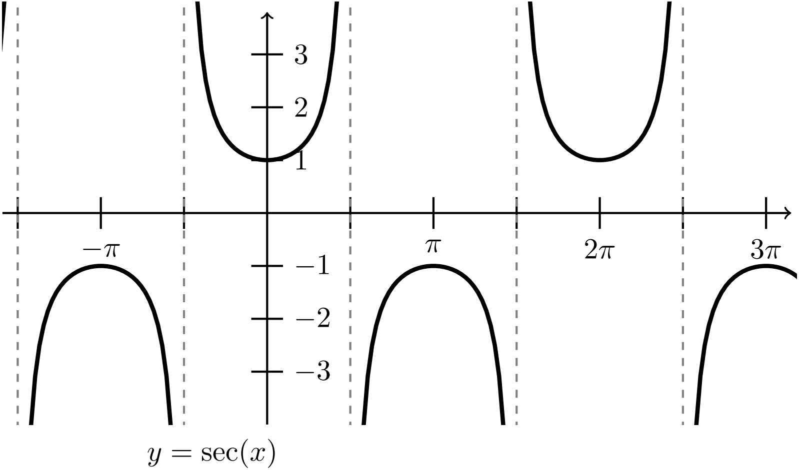

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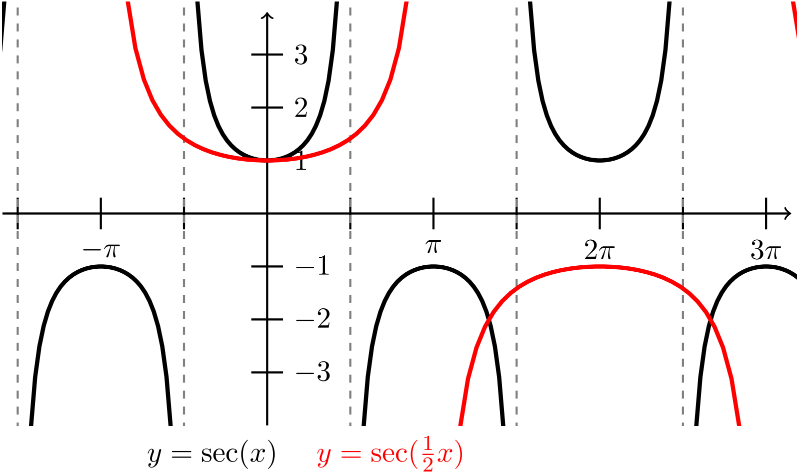

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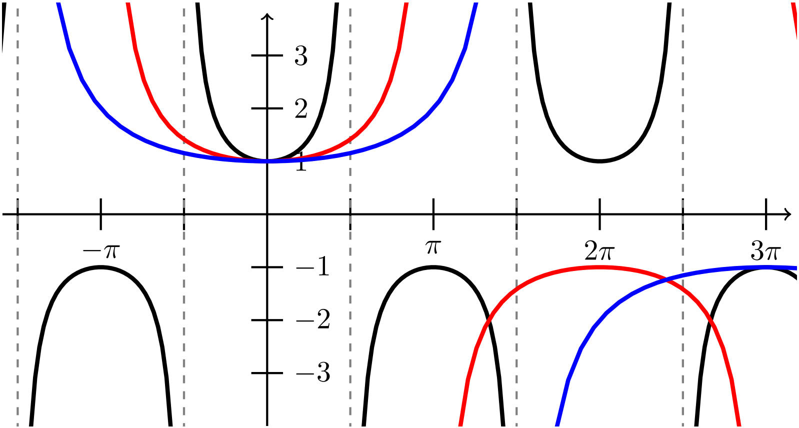


$y = \sec(x)$ $y = \sec(\frac{1}{2}x)$ $y = \sec(\frac{1}{3}x)$

# $b$ in $y = \csc(bx)$ or $y = \sec(bx)$

### What does $b$ do?

- Since original period of cosecant and secant is $2\pi$
- For $y = \csc(bx)$ or $y = \sec(bx)$, period is $\frac{2\pi}{|b|}$

# $b$ and $c$ together

$$y = \tan(bx + c)$$

$$y = \cot(bx + c)$$

$$y = \csc(bx + c)$$

$$y = \sec(bx + c)$$

# How to graph $y = \tan(bx + c)$

# How to graph $y = \tan(bx + c)$

Factor $b$

$$y = \tan\left[b\left(x + \frac{c}{b}\right)\right]$$

# How to graph $y = \tan(bx + c)$

Factor $b$

$$y = \tan\left[b\left(x + \frac{c}{b}\right)\right]$$

- period $= \frac{\pi}{|b|}$
- phase shift $= -\frac{c}{b}$

# How to graph $y = \tan(bx + c)$

Factor $b$

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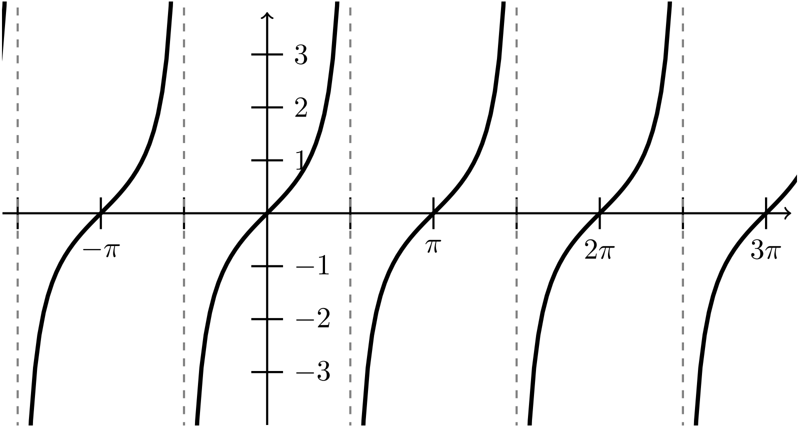


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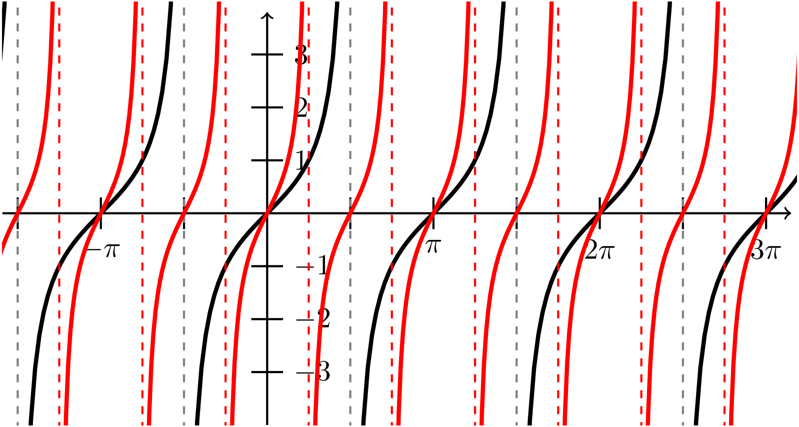


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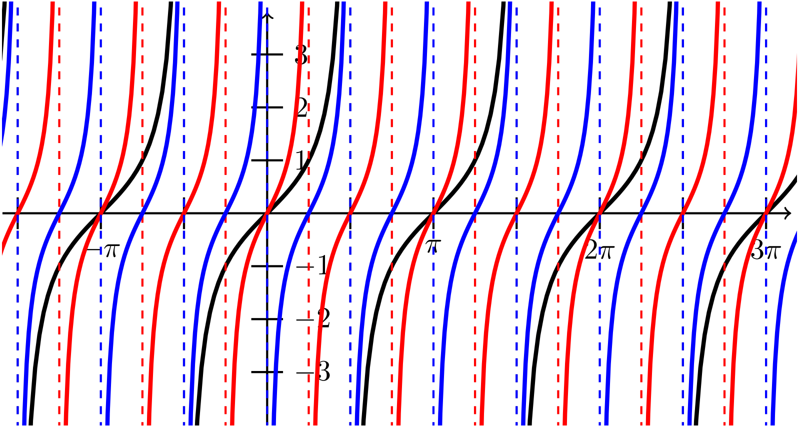
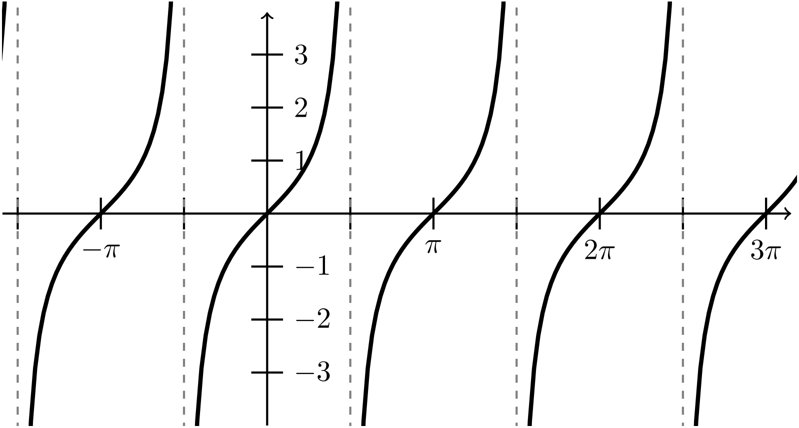



$y = \tan(x)$ $y = \tan(2x)$ $y = \tan(2x + \frac{\pi}{2})$

# Example: $y = \frac{1}{3}\tan(2x + \frac{\pi}{2})$

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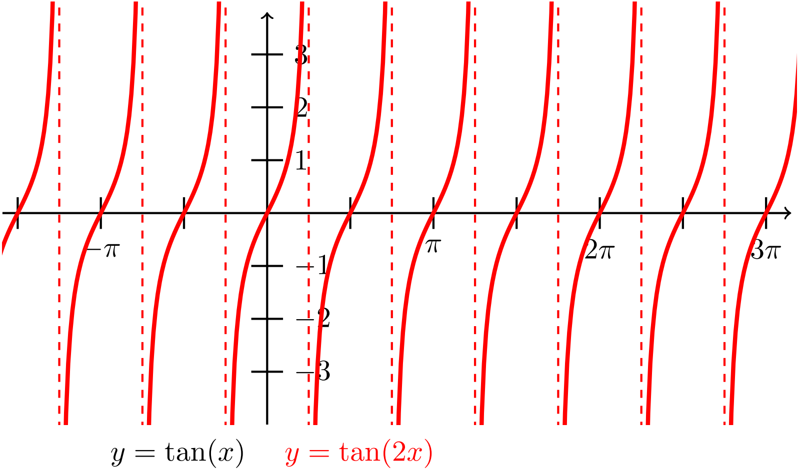

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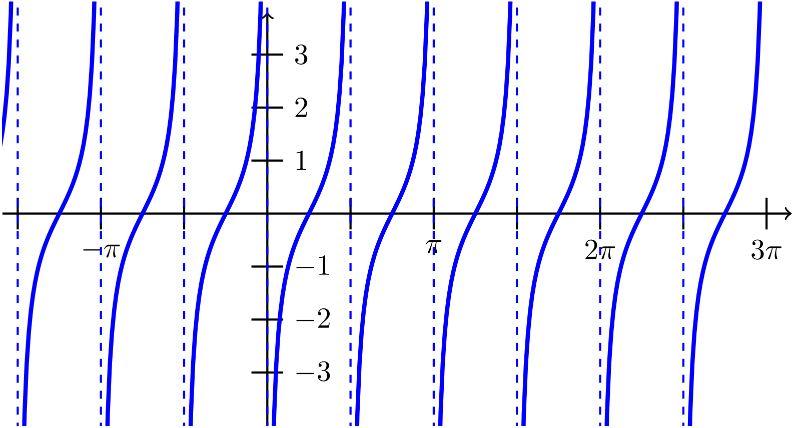


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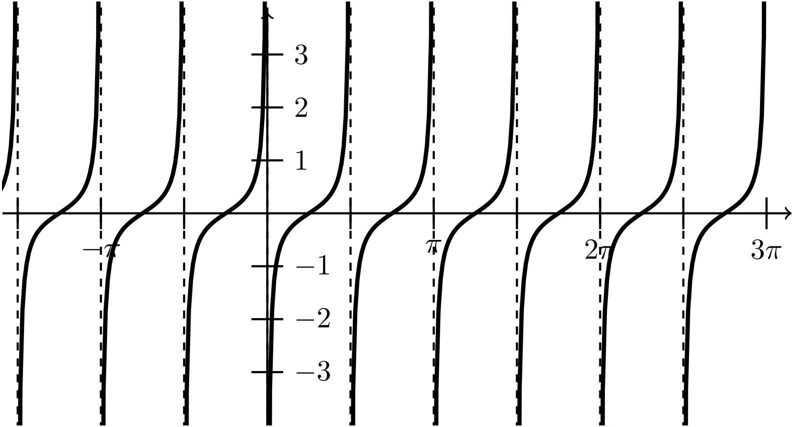


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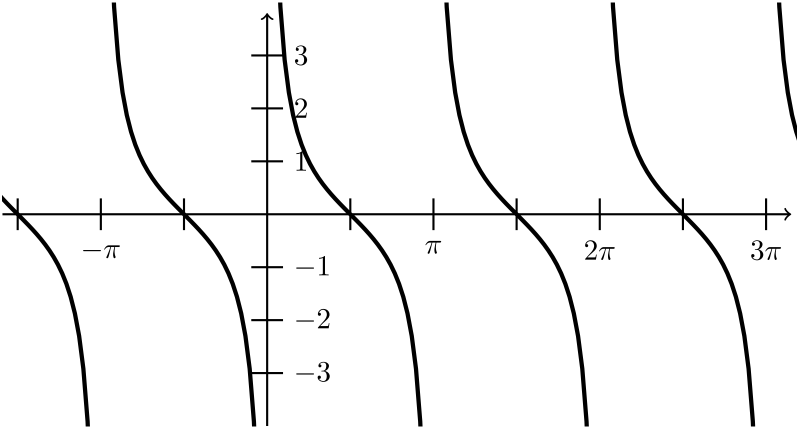


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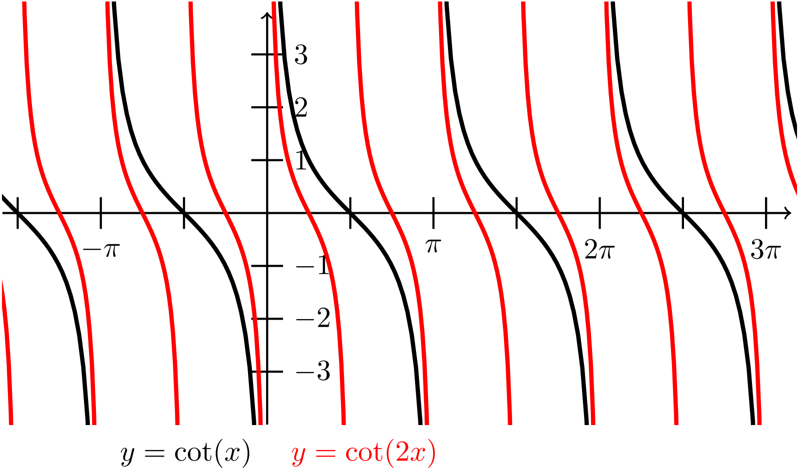

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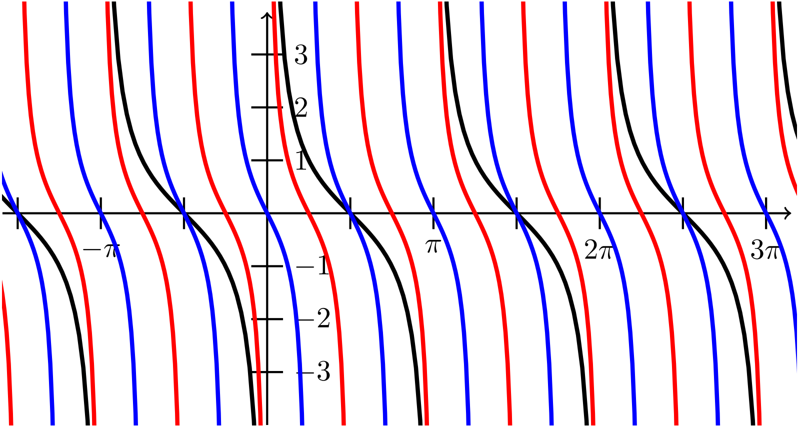


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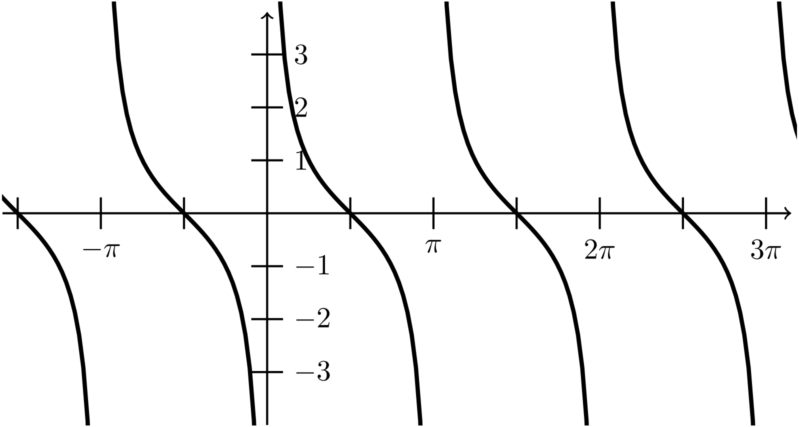


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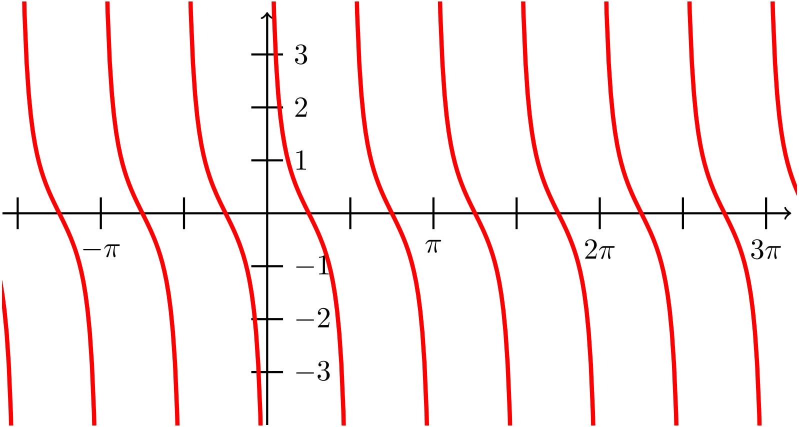


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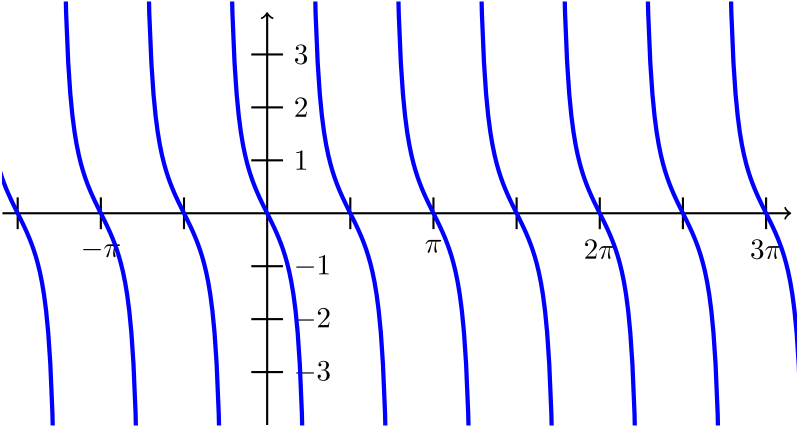


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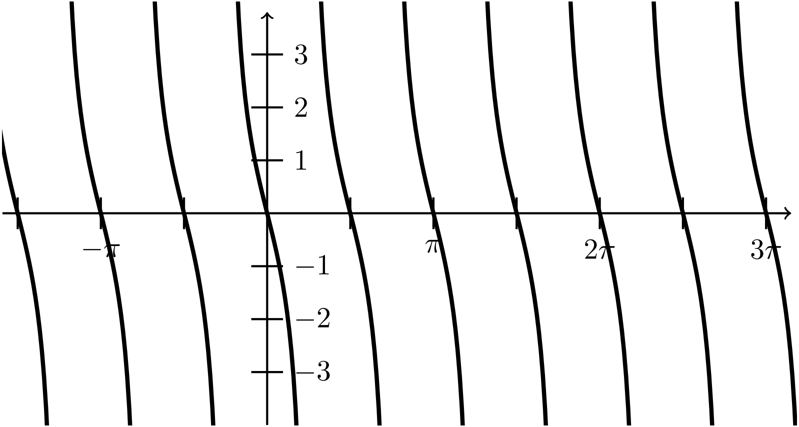


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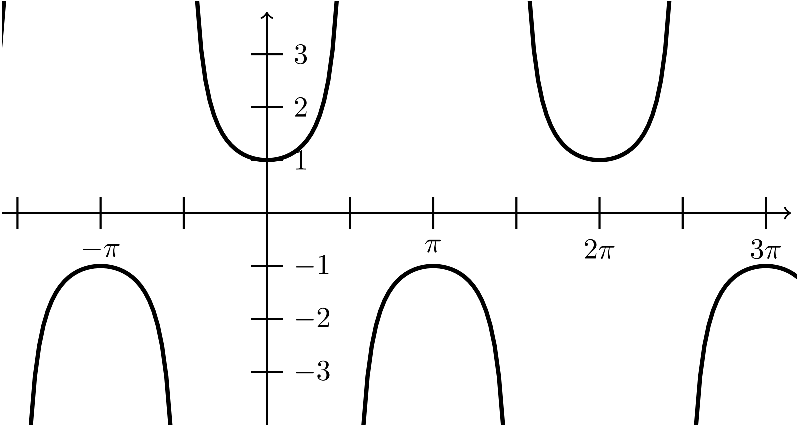


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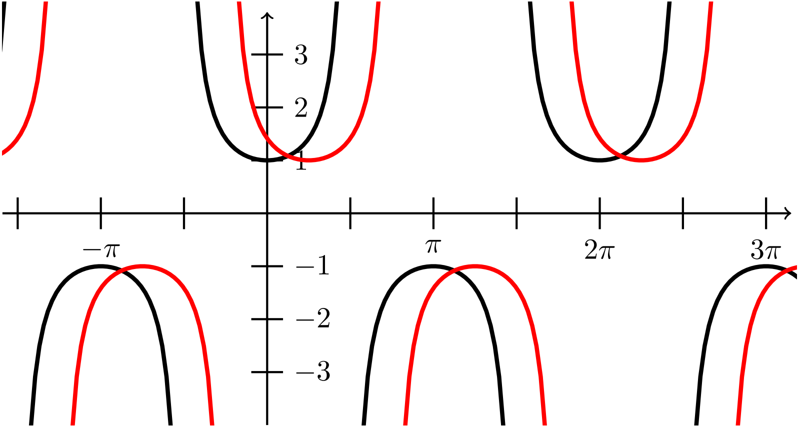


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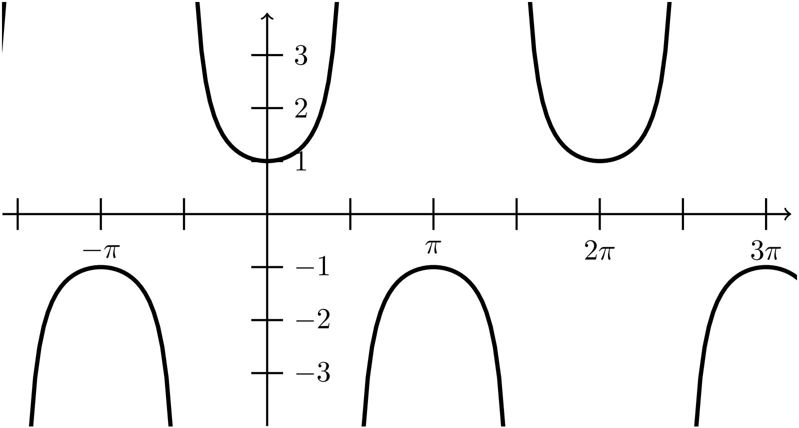


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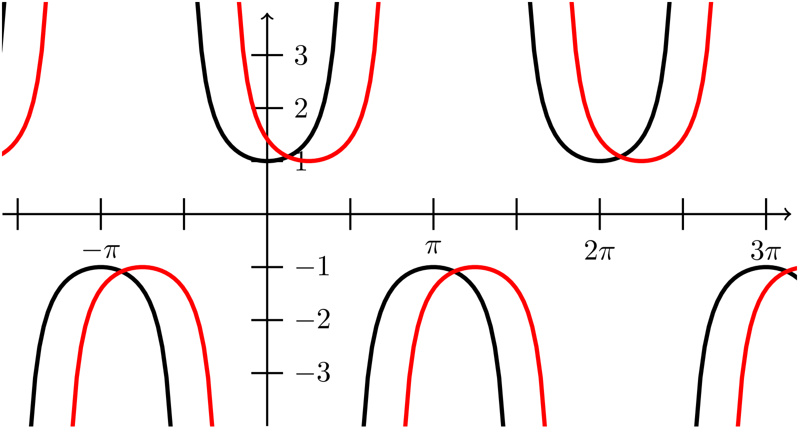


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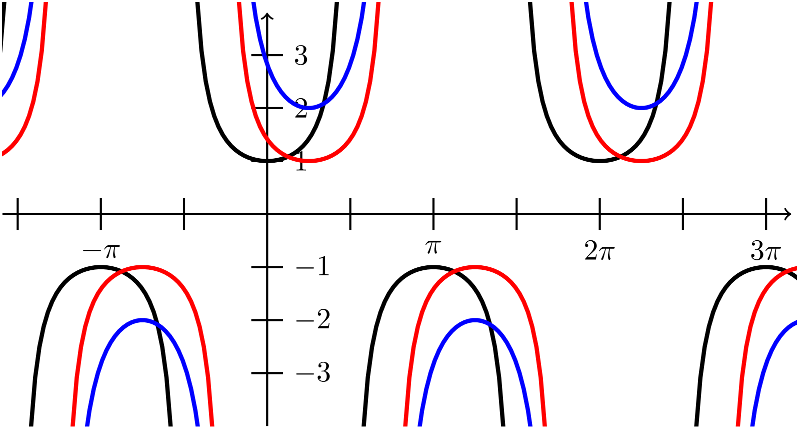


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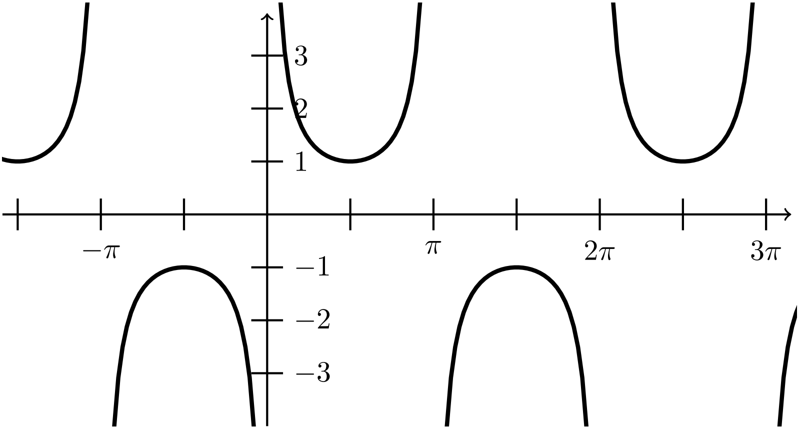


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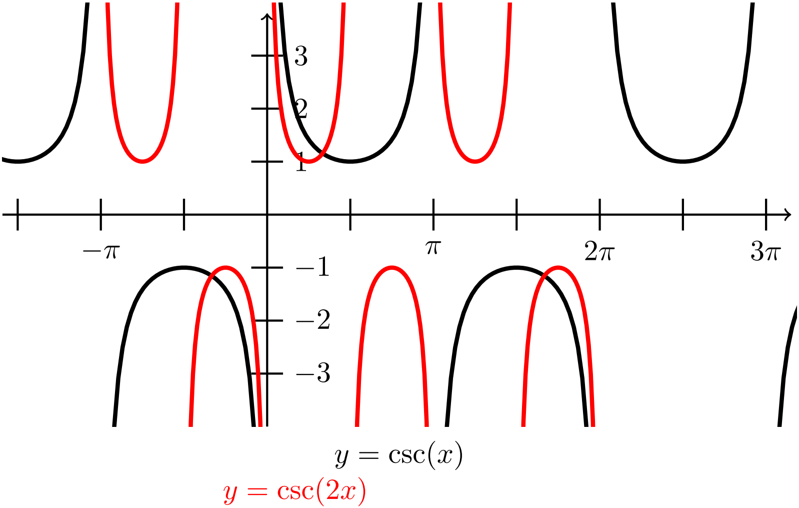

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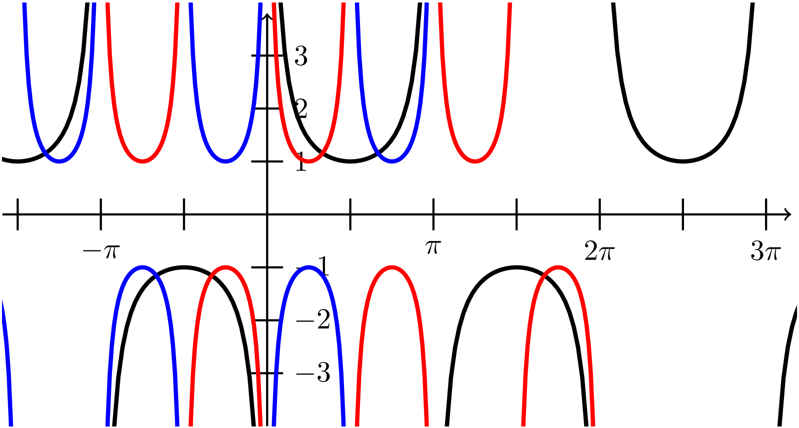


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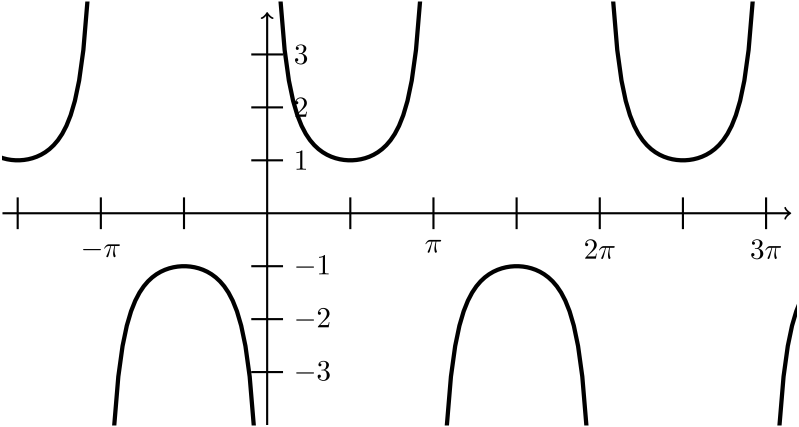
## Example: $y = \frac{1}{2}\csc(2x+\pi)$

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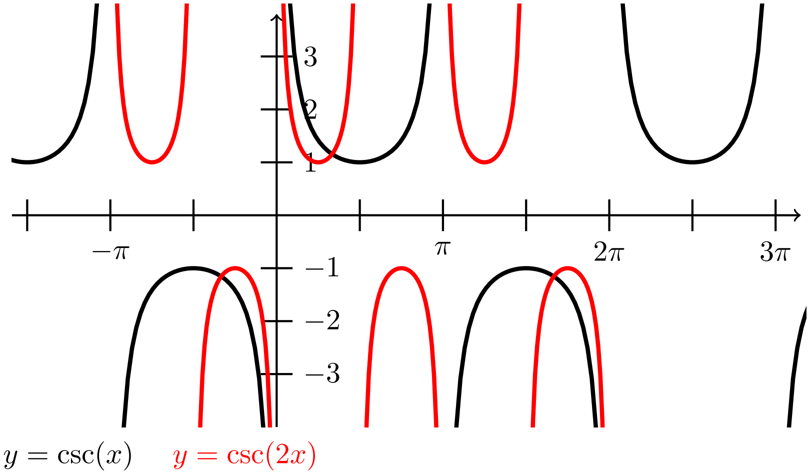

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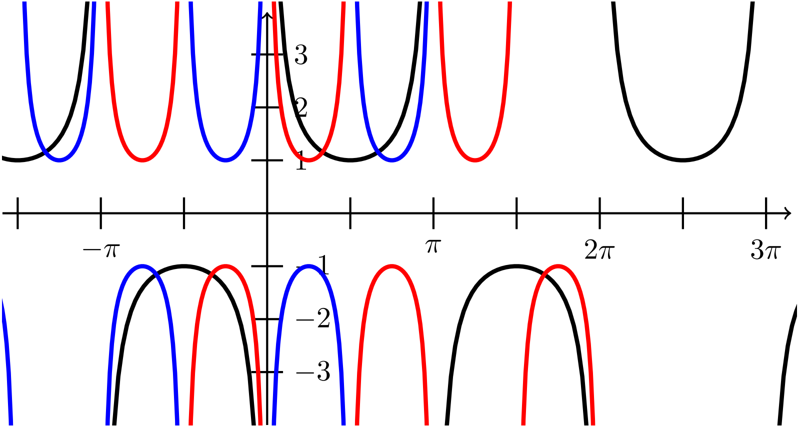


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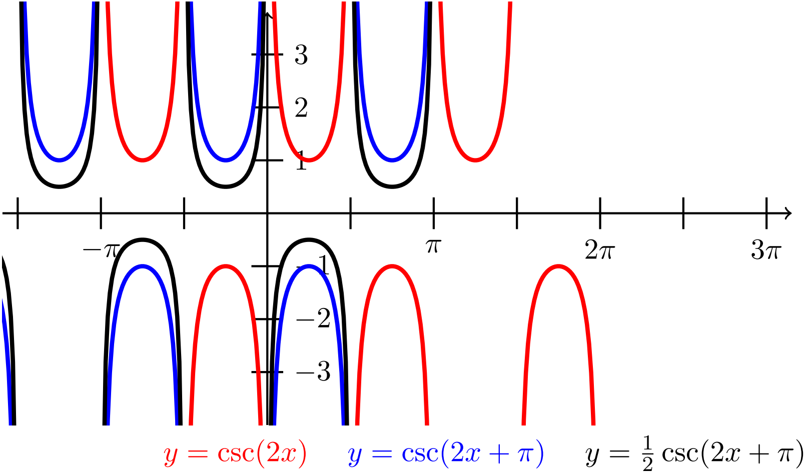
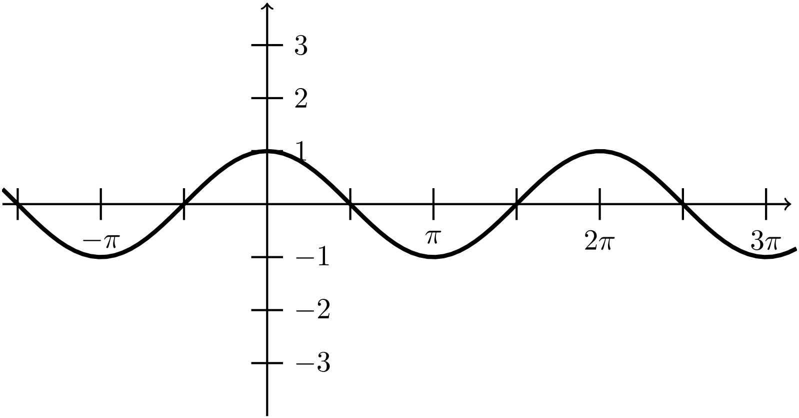


## More graph transformations

# $y = |\cos(x)| + 2$




$y = \cos(x)$, $y = |\cos(x)|$ and $y = |\cos(x)| + 2$

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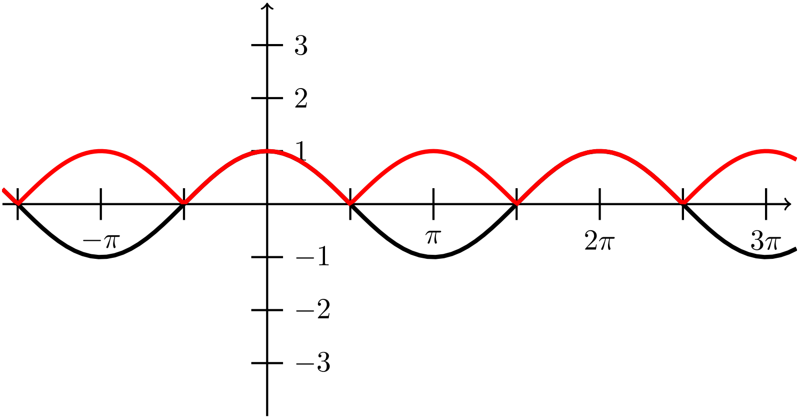


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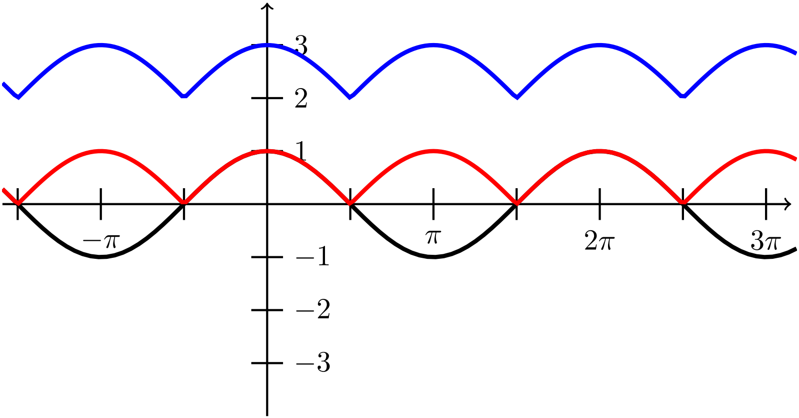
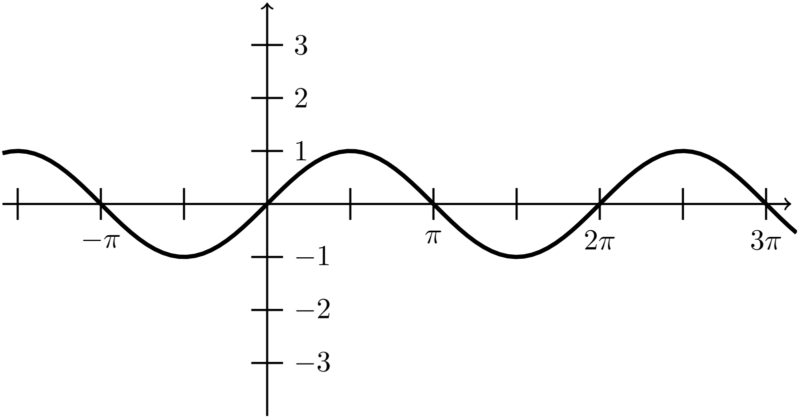



$y = \cos(x)$, $y = |\cos(x)|$ and $y = |\cos(x)| + 2$

# Recall $y = 2\sin(x)$



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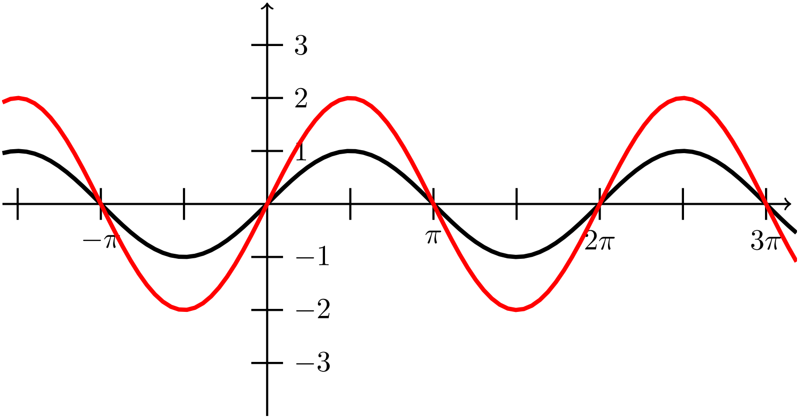


# What about $y = f(x)\sin(x)$?

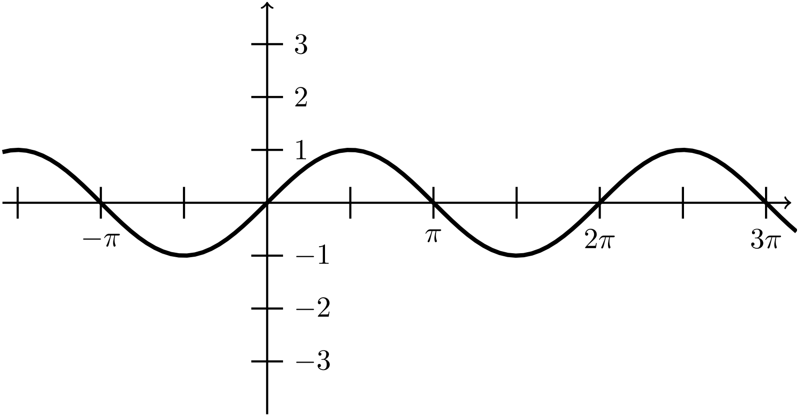
$f(x)$ is damping factor

# What about $y = f(x)\,\sin(x)$?

$f(x)$ is damping factor
"damped sine wave"

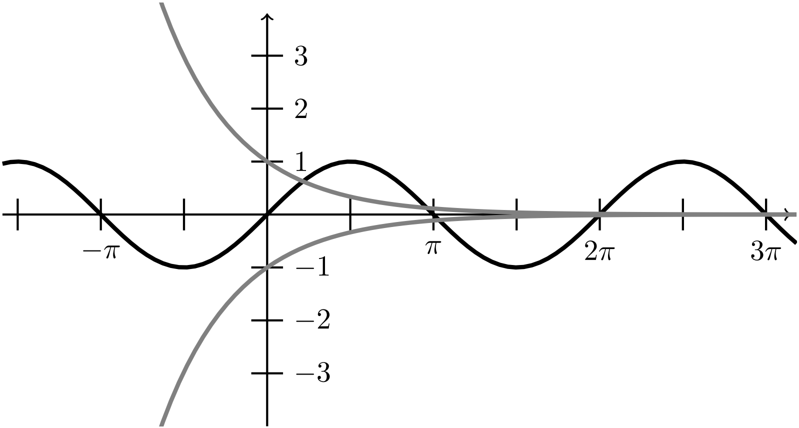
$$y = f(x)\,\sin(x)$$

# Recall $y = 2^{-x}\sin(x)$




In gray are $y = \pm 2^{-x}$

# Recall $y = 2^{-x}\sin(x)$

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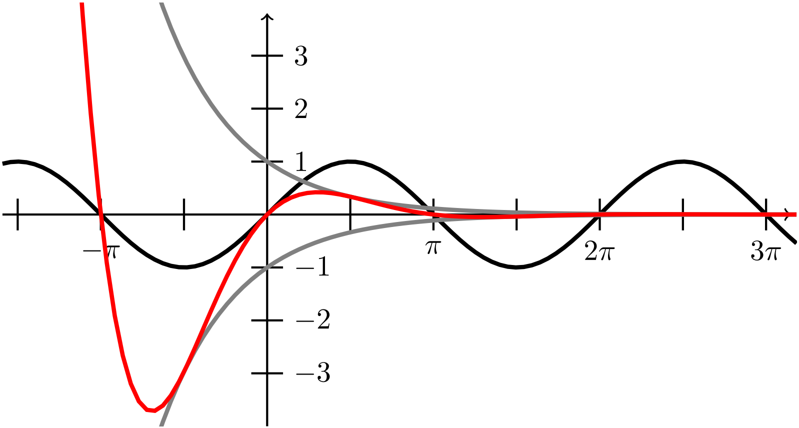


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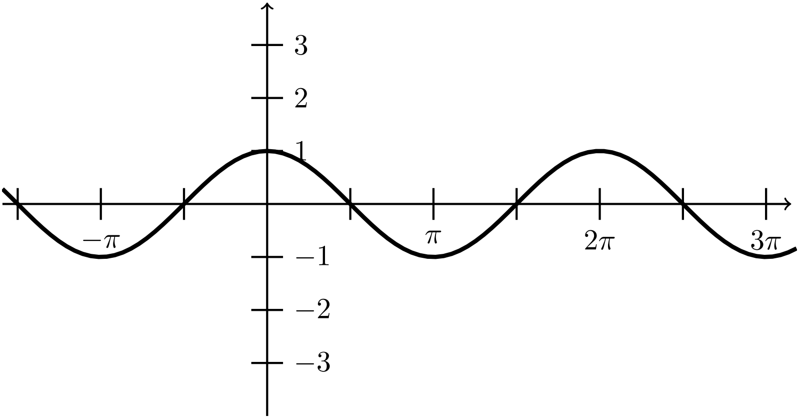
$y = \cos(x) + \sin(x)$



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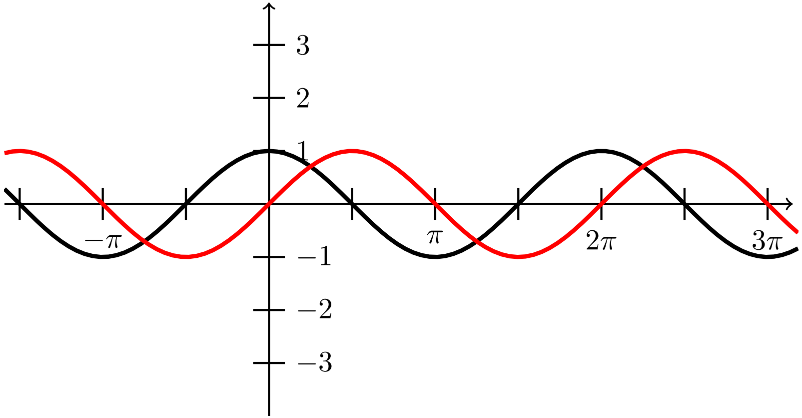

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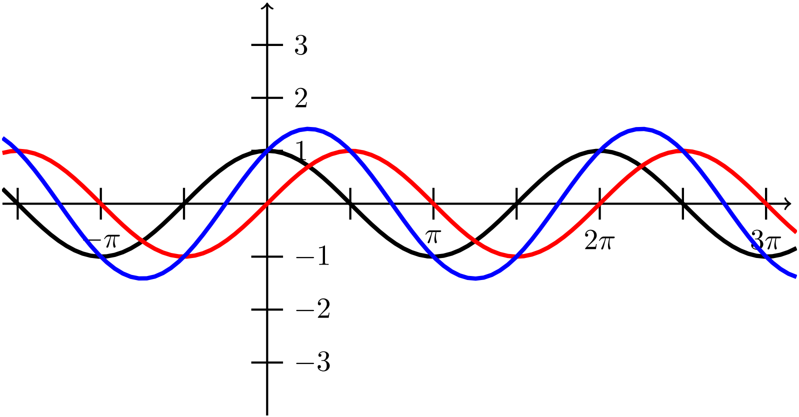
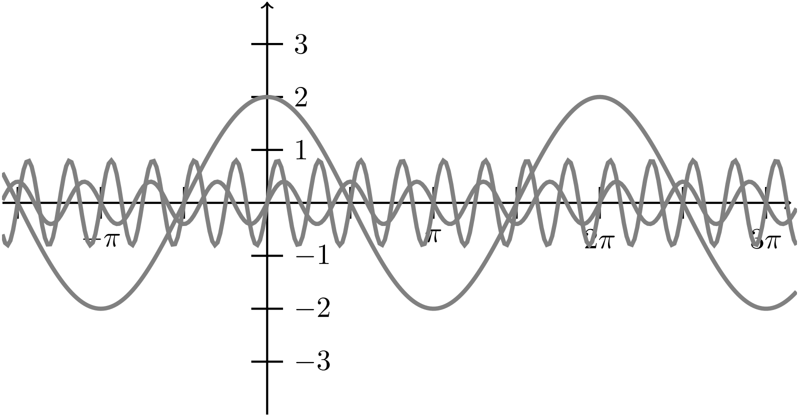


# Just for fun: $y = 2\cos(x) + .4\sin(5x) + .8\sin(8x)$




See http://method-behind-the-music.com/mechanics/physics
How do we instantly recognize the sounds of various musical instruments?

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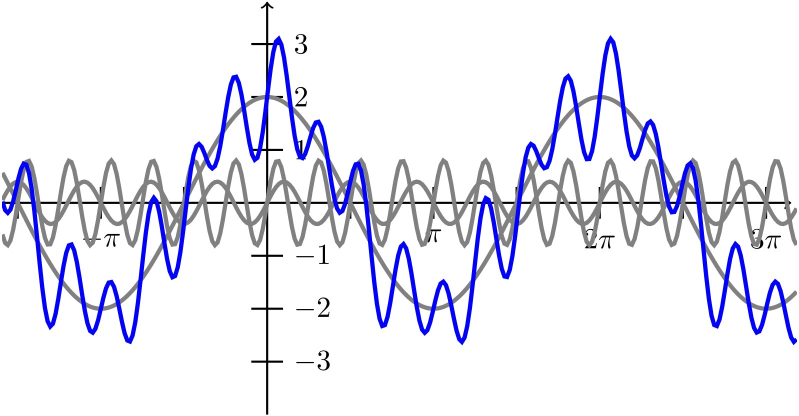


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