PHY 209
Space and Time in Elementary Physics

Euclidean Geometry and Non-Euclidean Geometry

Euclid was a Greek mathematician who lived around 300 BC. He wrote down a set of postulates (called the **Postulates of Euclidean Geometry**)

1. *Between any two points, a straight line segment can be drawn.*
2. *Any straight line segment can be extended indefinitely.*
3. *About any point, a circle of any radius can be drawn.*
4. *All right angles are equal.*
5. (The Parallel Postulate) *If two straight lines lying in the plane are met by another line, making the sum of the internal angles on one side less than 180 degrees, then those straight lines will meet on that side.* (This is related to a theorem in geometry which says that "*if a line intersects two parallel lines, the interior angles are supplementary*").

From these postulates, and a few minor others, comes all of the geometry that you learned in high-school.

One of the interesting things that comes up in geometry is this strange constant π. The first of our projects is to understand what π is.

What is π?

- Find a large flat area (*e.g.*, in the quad). You are going to measure the radius and the circumference of a large circle.
  - Pick a center for the circle and attach a rope to it. (You might want to have someone remain there holding the rope.)
  - With the other end of the rope in your hand, start walking away from the center, counting the number of paces until you get to the size you want. This quantity is the radius of that circle (measured in paces). Call this quantity r.

    $r =$

  - (The purpose of the rope is to keep you at a fixed distance r from the center as you walk around. After all, that *is* what a circle is.) Now, try to keep the rope taut and as low to the ground as you can. Pace a circle and count the number of paces needed to go around. This quantity is the circumference of that circle (measured in paces). Call this quantity C.

    $C =$

- Now, calculate

  the ratio between the circumference and twice-the-radius

  i.e., symbolically,

$$\frac{\text{Circumference}}{2\ (\text{radius})} = \frac{C}{2r}$$

$\frac{C}{2r} =$

- Repeat the above procedure for three more circles, each with a difference center and a different radius,

	circle 1	circle 2	circle 3
(r) radius	paces	paces	paces
(C) Circumference	paces	paces	paces
$\frac{C}{2r}$			

Study the values for $\frac{C}{2r}$ that you obtained for each circle. By the way, what are the units of the ratios $\frac{C}{2r}$ you calculated?

units?

If you did this carefully, you should have gotten roughly 3.14 (the exact value is 3.1415926535 ...) for each circle.

In a **Euclidean Geometry** (*i.e.*, when we are in a situation in which the above Postulates hold), the quantity

$$\begin{array}{c}\text{the ratio between}\\ \text{the circumference}\\ \text{and}\\ \text{twice-the-radius}\end{array} = \frac{\text{Circumference}}{2\ (\text{radius})} = \frac{C}{2r}$$

- is a **constant**, independent of the origin and the size of the circle.
- has a value of 3.1415926535 ...

This is **what π is.**

So, now can enter this first mathematical tool in our toolbox.
But part of learning about how something works is understanding where it fails to work and why.

Non-Euclidean Geometry

- Now, find a not-so-flat region... something curvy like a pitcher's mound or a bowl would be good. (There are some nice examples around campus and South Campus.) If you can't find one, you can use a basketball.
- Repeat the above procedure for constructing and pacing out these circles.

You should find that you do not get a constant ratio of 3.1415 ....

Why?

We are no longer dealing with a Euclidean Geometry. Instead, we have a non-Euclidean Geometry. One of Euclid's postulates does not apply here. Which one? It is the famous fifth postulate, The Parallel Postulate.

Think about the Earth. Have a friend and yourself stand on the equator, separated by some distance. For instance, have your friend stand 100 m to the east of you along the equator. You and your friend will walk due-North in straight lines (*i.e.*, along the lines of longitude). The equator is a line which crosses the two lines of your paths. Observe that the angle between your path and the equator is a right angle (90 degrees), as is the angle between your friend's path and the equator. Thus, the *sum of the interior angles* is 180 degrees, If the parallel postulate applied, then you and your friend would remain the same distance apart (100 m in our example) as you both travelled north. But what really happens? You get closer together as you travelled north. In fact, you meet at the north pole! (Note that the lines of longitude meet at the north pole.)

What is the lesson here?

The lesson is that the surface of the Earth has a non-Euclidean geometry. One way to say this is that: **the Earth is not flat.**

Does this mean that we can't really use the geometry we learned in high-school since we live on the Earth? No, it doesn't mean that. The Earth is approximately flat, like a plane surface, if one stays near to where one starts. So, if we keep our lines short, then we are okay using the Euclidean geometry from high-school. But if we want to use longer lines, we have to be careful... maybe even learn some non-Euclidean geometry.

• Can you think of any practical activity in which it is important to take the non-Euclidean geometry of the Earth into account?
Ans:

Bonus Question (1 pt):

A hunter travels 1 mi. South, 1 mi. East, then 1 mi. North (covering 3 mi. in total walking distance), and the hunter is back at the starting point.

- What color bear did the hunter shoot? Where did the hunter start?
- Are there any other places on the Earth's surface in which you can follow the same set of directions (in order) and end up at your starting point? *(If you get this part, you will impress me a lot.)*