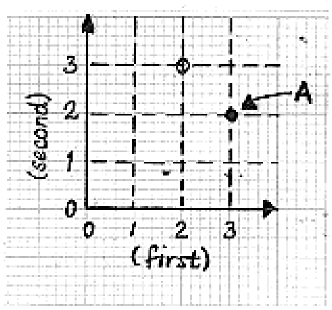
PHY 209
Space and Time in Elementary Physics

Cartesian Geometry

Graphs

Descartes was a French mathematician and philosopher in the 1600s. One of his contributions to mathematics was <u>**Coordinate Geometry**</u> The idea is that

- two intersecting directed-lines on a plane define a grid on the plane
- so that any point on the plane could be located by giving two numbers (in order) which locate it in the grid.

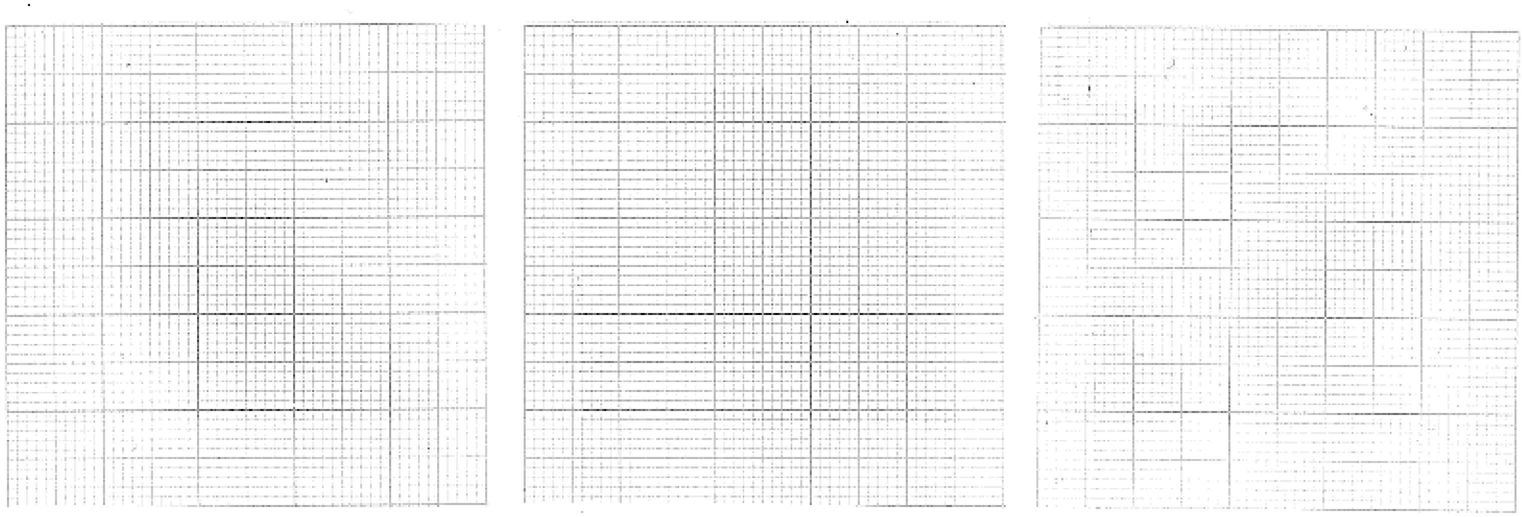


Each (ordered) pair of numbers are called <u>**coordinates**</u>. Each of these directed-lines is called a <u>**coordinate axis**</u>. The grid is called a <u>**coordinate system**</u>.

For instance, consider the grid on the left. The point A is located at $(3, 2)$. The first number gives the gridline along the first line. The second number gives the gridline along the second line. The order is important because $(2, 3)$ refers to another point.

For the following, use the graph below to plot each family of points:

- Plot the following points $(0, 2)$, $(-1, 2)$, $(-4, 2)$, $(3, 2)$. Lightly join the points with a smooth curve. (You should have gotten a straight line which is parallel to the x-axis.)
- Plot the following points $(3, 3)$, $(3, 4)$, $(3, -4)$, $(3, 1)$. Lightly join the points with a smooth curve. (You should have gotten a straight line which is parallel to the y-axis.)
- Plot the following points $(-3, -3)$, $(-1, -3)$, $(4, -3)$, $(2, -3)$. Lightly join the points with a smooth curve. (You should have gotten a straight line which is parallel to the x-axis.)



Functions

A **function** is: a pairing of two sets of numbers so that *"to each number in the first set, there corresponds exactly one number from the second set"*.

One way to represent a function is to use a **table** to list the pairs. Another way to represent a function is to use a **formula**. And, probably one of the best ways to represent a function is to use a **graph**, where the first set of numbers lies on the first axis (usually drawn horizontally) and the second set of numbers lies on the second axis (usually drawn vertically).

Consider the first *graph* you drew. The following *table* of pairs

first	second
0	2
−1	2
−4	2
3	2

represents the pairing *"give me any first number, and I will pair it up with the number 2"*. This satisfies the definition of a function because every first-number has exactly one second-number paired with it. It's okay that many first-numbers are paired up with the same number 2. That is allowed. The *formula* corresponding to this function can be written

$$y = 2.$$

[Sometimes one writes $y(x) = 2$ to remind us that *"y is a function of x"*.]

Consider the second *graph* you drew. The following *table* of pairs

first	second
3	3
3	4
3	−4
3	1

represents the pairing *"give me 3 for the first number, and I will pair it up with the numbers 3, 4, −4, and 1"*. This is **not** a function.

Why?

Ans:

- Is the third graph you drew the graph of a function? If so, write down in words what the pairing is and write a formula for the function.

It is an important skill to be able to look at a graph and to write down the corresponding formula.

- Plot the following points (-3,9), (-1,1), (0,0), (2,4), (4,16). Lightly join the points (in the order given) with a smooth curve. Is this a function? If so, try to write a formula for the function.

- Plot the following points (-1,3), (0,5), (1,7), (2,9). Lightly join the points with a smooth curve. Is this a function? If so, try to write a formula for the function.

- On the same graph, plot the following points (-1,-3), (0,-1), (1,1), (2,3). Lightly join the points with a smooth curve. Is this a function? If so, try to write a formula for the function.

- Plot the following points (9,-3), (1,-1), (0,0), (4,2), (16,4). Lightly join the points (in the order given) with a smooth curve. Is this a function? If so, try to write a formula for the function.

Constant Functions, Linear Functions, Quadratic Functions
Given a formula, one can always draw its graph.

> **If you don't know what a formula is trying to tell you, draw a graph! A graph is worth a thousand words.**

Three particularly important types of functions are

the Constant Functions $y = c$, where c is a constant

the Linear Functions $y = bx + c$, where b and c are constants

the Quadratic Functions $y = ax^2 + bx + c$, where a, b, c are constants

It should be clear that, *e.g.*, "a quadratic function with $a = 0$" is really a linear function.

- Plot the following **constant functions** on the same graph. [One way to do this is by plugging in a enough values of x and plotting the pair (x, y) until you get the sense of what the graph should look like.]
  $y = 0$
  $y = -3$
  $y = 4$
  In words, for a constant function $y = c$, what does c tell you?

- Plot the following **linear functions** on the same graph.
  $y = x + 1$
  $y = -x + 1$
  $y = 3x + 1$
  In words, for a linear function $y = bx + c$, what does b tell you?

- Plot the following **linear functions** on the same graph.
  $y = 2x$
  $y = 2x + 1$
  $y = 2x - 1$
  In words, for a linear function $y = bx + c$, what does c tell you?

- Plot the following **quadratic functions** on the same graph.

  $y = x^2$
  $y = -x^2$
  $y = 3x^2$

  In words, for a quadratic function $y = ax^2 + bx + c$, what does a tell you?

- Plot the following **quadratic functions** on the same graph.

  $y = x^2$
  $y = x^2 - 1$
  $y = x^2 + 1$

  In words, for a quadratic function $y = ax^2 + bx + c$, what does c tell you? (Hint: Look at what the graph is doing on the y-axis.)

- Plot the following **quadratic functions** on the same graph.

  $y = x^2 + 2x$
  $y = x^2 + 2x - 1$
  $y = x^2 + 2x + 1$

  In words, for a quadratic function $y = ax^2 + bx + c$, what does b tell you? (Hint: Look at what the graph is doing on the y-axis.)

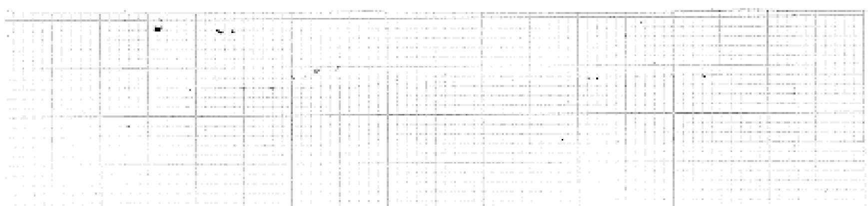
Experimental Data: Linear Kinematics

Here, we provide some important examples of these functions from the physical world. In class, there is an apparatus that will be used to demonstrate linear kinematics, *i.e.*, motion along a straight line.

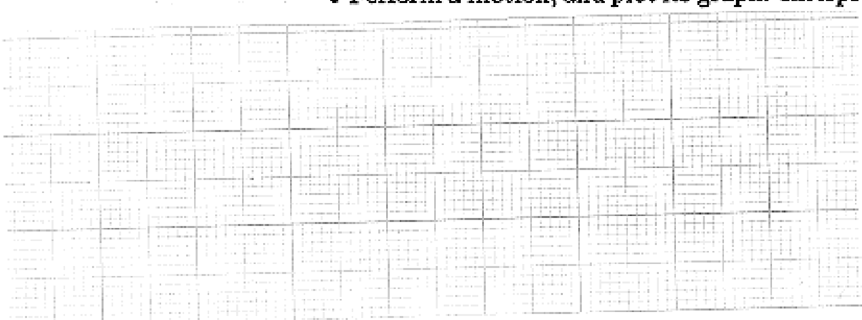
We will use this apparatus again when we discuss calculus later on in the course.

We will be interested in studying the position-of-the-object (call it x) as a function of t. In other words, for each time $t = t_1, t_2, \ldots$, we are interested in the corresponding positions $x_1 = x(t_1)$, $x_2 = x(t_2)$, ... *"You tell give me a time t, and I'll tell you the position-of-the-object, x.* [On a graph, we would draw the t-axis horizontally and the x-axis vertically.]

- Plot a graph of the following motion:
  - Traveling with constant velocity from $x = 1$ m to reach $x = 3$ m in the next 4 seconds
  - Remaining at $x = 3$ m for the next 2 seconds
  - Traveling with constant velocity back to $x = 1$ m in the next 2 seconds
  - Remaining at $x = 1$ m for the rest of the motion



- Perform a motion, and plot its graph. Interpret your motion in words.



Bonus Question (1 pt):

- An example of a **cubic function** is a function of the form $y = x^3 + ax^2 + bx + c$. Plot the following cubic functions on the same graph.

  $y = x^3 + 8$

  $y = x^3 - 8$

  Which one(s) cross(es) the x-axis? How many times?