# PHY 209
# Space and Time in Elementary Physics

## Vectors—Part I

### What is a vector?

A (Euclidean) <u>vector</u> is an object which

1. is added to other vectors using the **"Parallelogram rule"**
2. has a **magnitude** and a **direction**

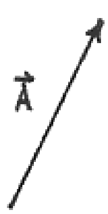
Vectors are usually written as $\vec{A}$ (with an arrowhead to remind us that this is a vector) or as $\mathbf{A}$ (as seen in textbooks).

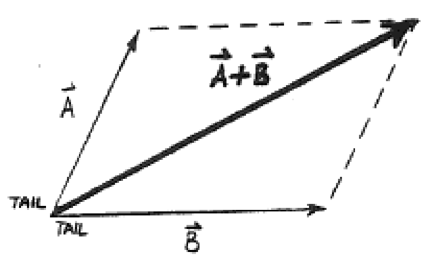
Vectors are pictorially represented by arrows, where the length and the direction of the arrow characterize the vector. (For now, we will regard two vectors as *"the same"* if they have the same magnitude and point in the same direction. This means that we can slide vectors around as long as we don't rotate them.)

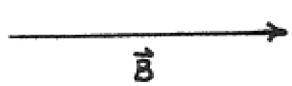
Textbooks have the luxury of fonts, whereas we, as handwriters, do not. It is a good habit to *always* use arrowheads to distinguish vectors from ordinary numbers.

**The Parallelogram Rule** is the addition-rule for vectors. There are two basic ways to think of this rule:

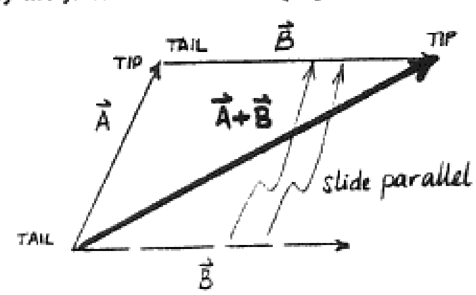
**parallelogram method:** To add vectors $\vec{A}$ and $\vec{B}$, place their *tails together* (sliding the vectors parallel, if necessary) and complete the parallelogram. The <u>vector sum</u> is another arrow drawn from *the tails of the original vectors to the opposite side of the parallelogram.*



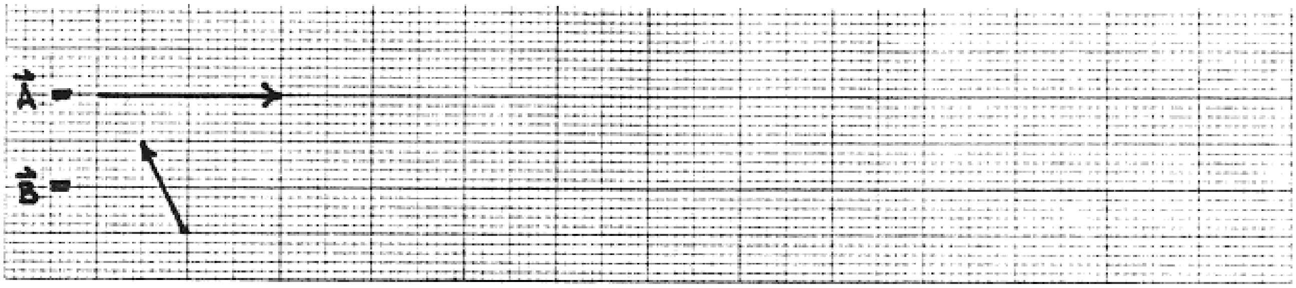




**tip-to-tail method:** To add vectors $\vec{A}$ and $\vec{B}$, start with one vector (say $\vec{A}$), then place the *tail* of the next vector ($\vec{B}$) *to the tip* of the first vector ($\vec{A}$) (sliding the vectors parallel, if necessary). Think of it as "taking a sequence of steps". The <u>vector sum</u> is another arrow drawn from *the tail of the first vector* to *the tip of the last vector.*

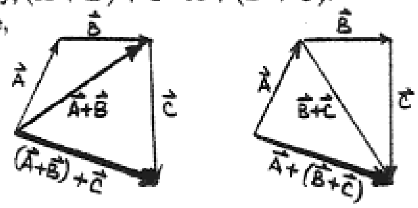


Note that vector-addition is **commutative**, *i.e.*, the order in which the vectors are added is irrelevant.
Algebraically, $\vec{A} + \vec{B} = \vec{B} + \vec{A}$.

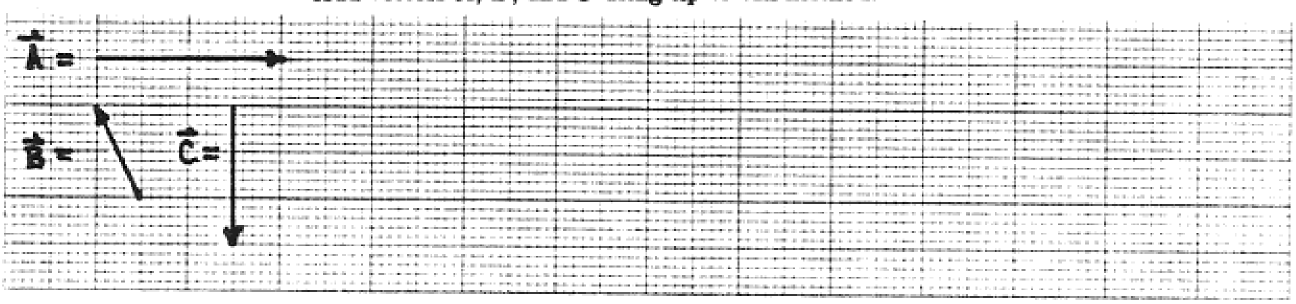
- **Add vectors $\vec{A}$ and $\vec{B}$ using both methods.**



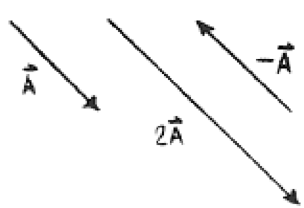

Vector-addition is also **associative**, *i.e.*, when adding three (or more) vectors together we can "add the vector-sum of the first-two to the third" or "add the first to the vector-sum of the last-two"
Algebraically, $(\vec{A} + \vec{B}) + \vec{C} = \vec{A} + (\vec{B} + \vec{C})$.
For instance,



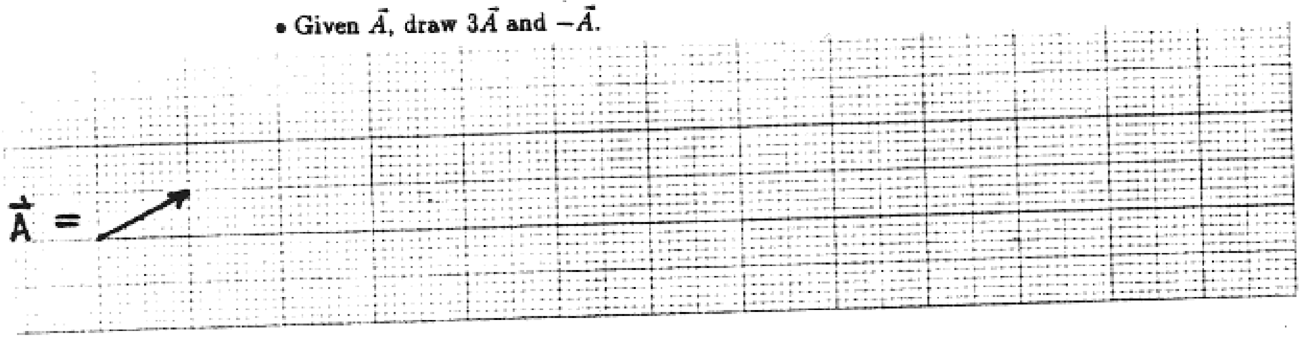

- **Add vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$ using tip-to-tail method.**




Vectors can also be *scaled* by multiplying them with numbers (called **scalars**). This is referred to as **scalar-multiplication**. Given a vector $\vec{A}$, the vector $2\vec{A}$ is another vector which points in the same direction as $\vec{A}$, but is twice as long as $\vec{A}$. Given a vector $\vec{A}$, the vector $(-1)\vec{A}$ (or $-\vec{A}$) is another vector which points in the opposite direction of $\vec{A}$, but is as long as $\vec{A}$.
For instance,



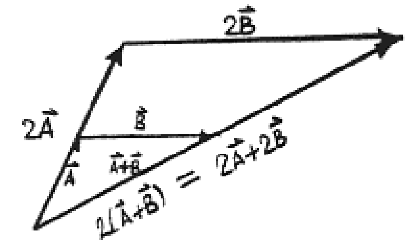
• Given $\vec{A}$, draw $3\vec{A}$ and $-\vec{A}$.



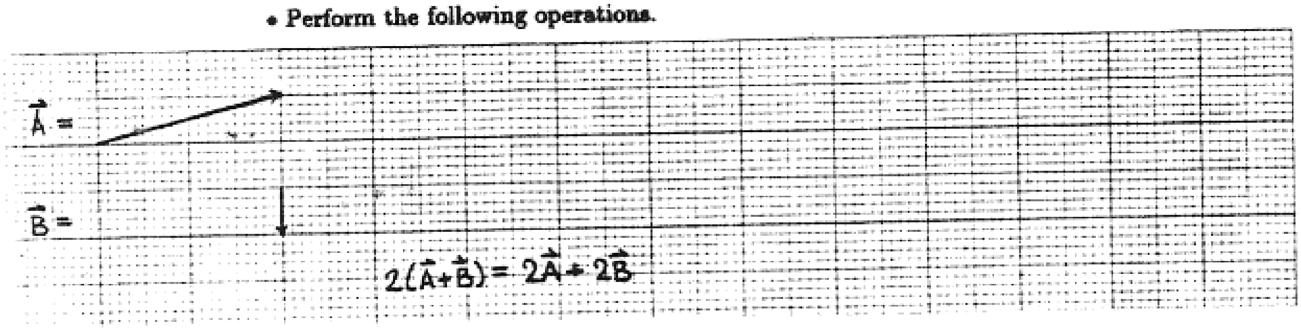

Scalar-multiplication is <u>**distributive**</u> in the sense that:

$$(k+l)\vec{A} = k\vec{A} + l\vec{A}, \qquad \text{and } k(\vec{A}+\vec{B}) = k\vec{A} + k\vec{B}.$$

For instance,




• Perform the following operations.




**Coordinates as "Crutches"—Part I**

In your diagrams, we used graph-paper to help us draw the vectors neatly and accurately. (You probably counted boxes to see how to draw vectors.) But it is an important lesson to realize that the graph-paper is only a crutch... and that we can perform all of the operations described above without graph-paper. We really only need a ruler.

One result of this is that it does not matter, *e.g.*, how our graph-paper is oriented. The result of vector-addition will be the same, *i.e.*, adding $\vec{A}$ and $\vec{B}$ will result in the same vector (with the same magnitude and direction), regardless of how the graph-paper is oriented.

• Perform the following operations.