PHY 209
Space and Time in Elementary Physics

Vectors—Part IV (an application: Force)

Let's consider an application of vectors in elementary physics: the force.

Intuitively, a <u>force</u> applied on an object is "a push or a pull" on that object. The magnitude of the force is "the strength of the applied force". The direction of the force is "the direction in which the push or pull is being applied".

If two (or more) forces are simultaneously applied on an object, one is often interested in the total force (or "net-force") being applied to that object. That is to say, one is often interested in a single "resultant-force" which produces that same result as those two (or more) forces. Experiment shows that this net-force is given by the vector-sum of all of the applied force vectors.

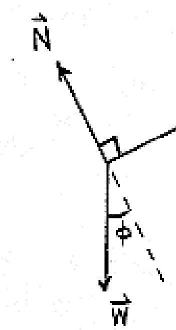
In other words, <u>**force** *is a physical quantity described by a* **vector**.</u>

Later on, we will learn about "the Newton Laws of Motion" which relate the forces applied on an object to the state of motion of that object. This is "where the physics is". Once the two quantities are related, we then have a purely mathematical problem to solve.

For now, we will only be concerned the following types of mathematical problems:

- Given a set of known vectors, find the vector-sum.
- Given a set of vectors—some known, some unknown—and given the vector-sum, determine the unknown vectors.

Consider the following set of vectors.

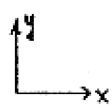


By now, you should know how to compute $\vec{N} + \vec{T} + \vec{W}$ geometrically—either by the tip-to-tail method or by repeated application of the parallelogram-rule. If a set of coordinate-axes are introduced and you are given the x- and y-components of each vector, then you should be able to easily compute the vector-sum algebraically.

Sometimes, however, the x- and y-components (with respect to some set of axes) are not presented to you on a platter. Instead, you are often presented with a diagram (which gives you information about the *directions* of some vectors) and you are told the *magnitudes* of some vectors. You need to do some work to express the vectors in terms of components.

Remember, you can choose any set of axes to help you evaluate the vector-sum. Some choices of axes will make the calculation easier than other choices.

Let's choose the following set of axes.
Then



$$\begin{aligned}
\vec{F} &= \vec{N} + \vec{T} + \vec{W} \\
&= (N_x\hat{\imath} + N_y\hat{\jmath}) + (T_x\hat{\imath} + T_y\hat{\jmath}) + (W_x\hat{\imath} + W_y\hat{\jmath}) \\
&= (N_x + T_x + W_x)\hat{\imath} + (N_y + T_y + W_y)\hat{\jmath} \\
&= (F_x)\hat{\imath} + (F_y)\hat{\jmath}
\end{aligned}$$

The "components of the vector-sum" is the "sum of the vector-components".

Using the counterclockwise angle drawn from the positive-x axis to each vector,

$$\begin{aligned}
F_x &= N\cos(90 + \phi) + T\cos(\phi) + W\cos(270) \\
F_y &= N\sin(90 + \phi) + T\sin(\phi) + W\sin(270)
\end{aligned}$$

If one makes use of some identities in trigonometry:

$$\cos(90+\phi) = -\sin(\phi) \qquad \sin(90+\phi) = \cos(\phi)$$

which are specific cases of

$$\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$$

$$\sin(A+B) = \sin(A)\cos(B) + \cos(A)\sin(B),$$

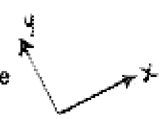
we can write

$$\begin{aligned} F_x &= -N\sin(\phi) + T\cos(\phi) \\ F_y &= N\cos(\phi) + T\sin(\phi) - W \end{aligned}$$

You could have written down this set of equations immediately by doing **trigonometry** and paying close attention to the signs, which you can read-off from the diagram. Don't mess-up the various sin and cos, and don't mess up the signs! *This is the price one pays if one insists on working with only acute angles.*

Use whichever method you are more comfortable with.

I just want you to appreciate that the first set is very easy to write down if you are able to identify the counterclockwise angle drawn from the positive-x axis (*i.e.*, do **arithmetic**). All of the various signs and "sin and cos" are already taken care of. *The price one pays here is that one has to know the* sin *and* cos *of angles that are not necessarily acute.*



Instead of the first choice of axes above, use the following set of axes. (The x-axis is parallel to $\vec{T}$, and the y-axis is parallel to $\vec{N}$.)

- [•] Find the x-component for each of $\vec{N}$, $\vec{T}$, $\vec{W}$.
  Find the x-component, F_x, of the vector-sum $\vec{N}+\vec{T}+\vec{W}$.

- [•] Find the y-component for each of $\vec{N}$, $\vec{T}$, $\vec{W}$.
  Find the y-component, F_y, of the vector-sum $\vec{N}+\vec{T}+\vec{W}$.

Note that a little algebra will show that "although the values for F_x and F_y depend on the choice of axes, the value of $(F_x)^2+(F_y)^2$ does not depend on the choice of axes.

- [•] Suppose that $\vec{N}+\vec{T}+\vec{W}=\vec{0}$. Suppose that W (the magnitude of $\vec{W}$) and ϕ are known. Express T (the magnitude of $\vec{T}$) and N (the magnitude of $\vec{N}$) in terms of W and ϕ. (As a check: If $W = 100$ and $\phi = 30°$, then $T = 50.0$ and $N = 83.3$.)