PHY 209
Space and Time in Elementary Physics

Some Fundamental Constants of Physics

G

Historically, **Galileo** and **Newton** are regarded as the founders of "classical physics". The laws of motion that they wrote down were (and have been) adequate as descriptions of everyday physics.

Galileo's **Law of Falling Bodies** (1638) is the observation that

$g = 9.8\ \mathrm{m/s^2}$

$$\vec{a} = \vec{g}$$

> *"All bodies near the earth's surface fall under the influence of gravity with the same acceleration $\vec{g}$ (neglecting air resistance)".*

Newton extended this to **The Law of Universal Gravitation** (1687):

$$\vec{F}_{\mathrm{grav}} = -G\frac{Mm}{r^2}\hat{r}$$

> *"The force $\vec{F}_{\mathrm{grav}}$ that a point mass m feels due to another point mass M has magnitude proportional to the product of their masses, and inversely proportional to the square of their separation distance."*

The constant of proportionality is denoted by G, and is called the **Newtonian gravitational constant**. It has a value of $6.672 \times 10^{-11}\ \mathrm{Nm^2/kg^2}$. The vector $\hat{r}$ is the unit-vector drawn from the source-mass M to the mass m. The minus sign tells us that the force is in the opposite direction of $\hat{r}$. In physical terms, this says that mass m is *gravitationally-attracted* to mass M. Since masses are never negative, this says that **gravity is an attractive force** between any two masses.

good rule of thumb: $G \approx \frac{2}{3} \times 10^{-10}\ \mathrm{Nm^2/kg^2}$

- Let M_{earth} be the mass of the Earth.
  Suppose that a small stone (with mass m) is set at a height h ($\ll R_{\mathrm{earth}}$) above the ground (*i.e.*, "near the Earth's surface").

  Using the above two laws along with Newton's Law of Motion, $\vec{F} = m\vec{a}$, derive an algebraic expression for the acceleration due to gravity g in terms of the gravitational constant G and the mass M_{earth} and radius R_{earth} of the earth.

- Using your formula and the "old definition of the meter", obtain a numerical value for the mass M_{earth} of the earth.

good rule of thumb: $10^7\ \mathrm{m} \approx \frac{1}{4}(2\pi R_{\mathrm{earth}})$

From mathematical principles, Newton developed his laws of motion. He found that his *theoretical predictions* were in good agreement with *experimental data* taken by other people before him (like Galileo and Kepler). However, it wasn't until 1798 that an accurate value for Newton's constant G was experimentally measured by **Cavendish.** Cavendish determined G from careful measurements of the attraction between two masses in his lab. In fact, at the time, only the acceleration due to gravity g (from Galileo's experiments) and the radius of the earth R_{earth} (from astronomical observations) were known. So, in light of the expression obtained on the previous page, it is often said that Cavendish "weighed the earth" by accurately determining G from within in his lab.

e

Earlier, Cavendish (1772) and Coulomb (1785), each independently, used a similar apparatus to measure the attraction between two electrically-charged objects. Their experiments had led them to Coulomb's <u>Law of Electrostatics</u> (1785):

$$\vec{F}_{\text{elec}} = k\frac{Qq}{r^2}\hat{r}$$

> *"The force $\vec{F}_{\text{elec}}$ that a point charge q feels due to another point charge Q has magnitude proportional to the product of their charges, and inversely proportional to the square of their separation distance."*

The constant of proportionality is denoted by $k = \frac{1}{4\pi\epsilon_0}$. ϵ_0 is called the **permittivity constant**, which has a value of $8.854 \times 10^{-12}\ \text{C}^2/\text{Nm}^2$. The vector $\hat{r}$ is the unit-vector drawn from the source-charge Q to the charge q. Since charges can be positive or negative or zero, this says that **oppositely-signed charges attract, like charges repel**, and **only charged particles interact electrostatically**.

good rule of thumb: $\frac{1}{4\pi\epsilon_0} \approx 9 \times 10^9\ \text{Nm}^2/\text{C}^2$

good rules of thumb: $m_e \approx 9 \times 10^{-31}$ kg $q_e \approx \frac{1}{6} \times 10^{-18}$ C.

- The mass of an electron is $m_e = 9.109 \times 10^{-31}$ kg.
  The charge of an electron is $q_e = 1.602 \times 10^{-19}$ C.
  Suppose that two electrons are separated by a distance $r = 1$ m.
  Calculate the gravitational attraction F_{grav} between these two electrons.
  Calculate the electrostatic repulsion F_{elec} between these two electrons.
  What is the ratio between their strengths $\frac{F_{\text{grav}}}{F_{\text{elec}}}$?

c

Galileo tried but failed to measure the speed at which light travels. In his experiments, light traveled from one point to another almost instantaneously.

The first measurement of the speed of light was done by **Roemer** (1675), who studied the sunlight reflecting off the moons of Jupiter. His observations led him to conclude that light does not travel infinitely fast from point to point, but that it travels with a finite speed. More refined experiments have since determined that .

good rule of thumb: $c \approx 3 \times 10^8$ m/s

The speed of light is finite,
and has a measured value of $c = 2.9979 \times 10^8$ m/s.

Recently, the speed of light was ***defined***, promoting it to be a new standard by which the old standards are defined. For example, the meter is no longer the length of a certain metal bar in France, but is the distance that light travels in a specifically chosen time. This is an example of a distance measured in "light-units".

- How far does light travel in 1 ns $= 1 \times 10^{-9}$ s?
  Give a practical example of that length.

- How far does light travel in 1 s?
  Give a practical example of that length.

- How far does light travel in 1 min = 60 s?
  Give a practical example of that length.

- How far does light travel in 1 hour = 3600 s?
  Give a practical example of that length.

good rule of thumb: $1\text{yr} \approx \pi \times 10^7$ s

- How many seconds are in a year?
  How far does light travel in 1 year?
  Give a practical example of that length.

The speed of light turns out to be one of the most important constants of physics. In 1860, **Maxwell** made a remarkable discovery with his new contribution to the laws of electricity and magnetism. Up to that time, the laws of magnetism involved another physical constant μ_0 called the **permeability constant**, which has a value of 1.26×10^{-6} H/m. It turns out that when all of those laws of electricity and magnetism are taken together with Maxwell's new contribution, those two constants ϵ_0 and μ_0 combined in a peculiar combination:

good rule of thumb: $\mu_0 \equiv 4\pi \times 10^{-7}$ H/m

$$\frac{1}{\sqrt{\epsilon_0 \mu_0}}$$

- It turns out that this has the units of a speed.
  Calculate the numerical value of this quantity.

This remarkable discovery was a very important contribution to our understanding of the physical world. It told us that electricity and magnetism and optics(!), once separate fields of study, were somehow deeply related to each other. In fact, this contributed to **Einstein**'s development of the Theory of Relativity (1905).