PHY 209
Space and Time in Elementary Physics

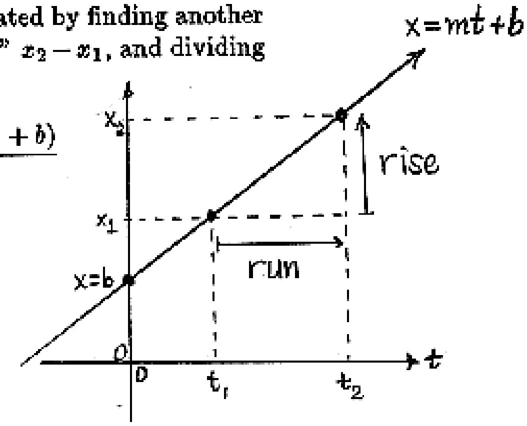
Linear Approximations—Prelude to Differential Calculus

The Slope

Recall that the "**slope** m of a line" is a measure of the **rate-of-change** of the linear function $x = mt + b$ with respect to the independent "running" variable t. (Since this is a line, the slope m is a constant. The "intercept b" is the value of the function when $t = 0$; it is always a constant.)

There is a useful mnemonic: "**slope**" is "**rise by run**". Given a point (x_1, t_1) on the line $x = mt + b$, the slope m can be calculated by finding another point (x_2, t_2) on the same line, then calculating the "rise" $x_2 - x_1$, and dividing it by the "run" $t_2 - t_1$:

$$\frac{\text{rise}}{\text{run}} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{(mt_2 + b) - (mt_1 + b)}{t_2 - t_1} = \frac{m(t_2 - t_1)}{t_2 - t_1} = m$$



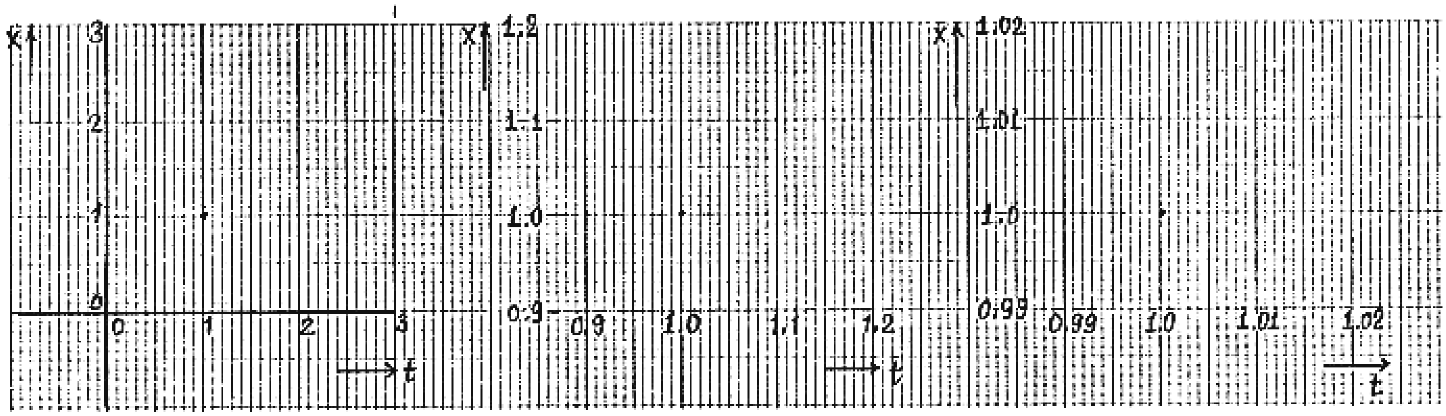
"Zooming In"

We wish to study the behavior of functions as we "zoom in" on them.

Fill in this chart:

$(t+h)$	t	h	$= t+h$
$t = 1,\ h = 1$			
$t = 1,\ h = 0.1$			
$t = 1,\ h = 0.01$			

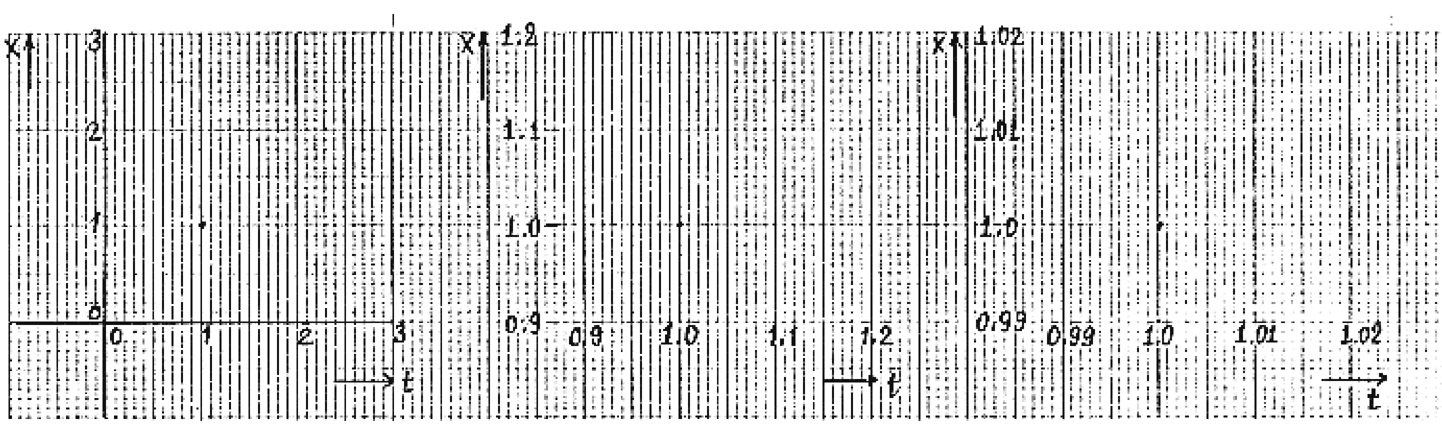
Plot the function $x = t$ on each of the graphs below:



Fill in this chart:

$(t+h)^2$	t^2	$2ht$	h^2	$= t^2 + 2ht + h^2$
$t = 1,\ h = 1$				
$t = 1,\ h = 0.1$				
$t = 1,\ h = 0.01$				

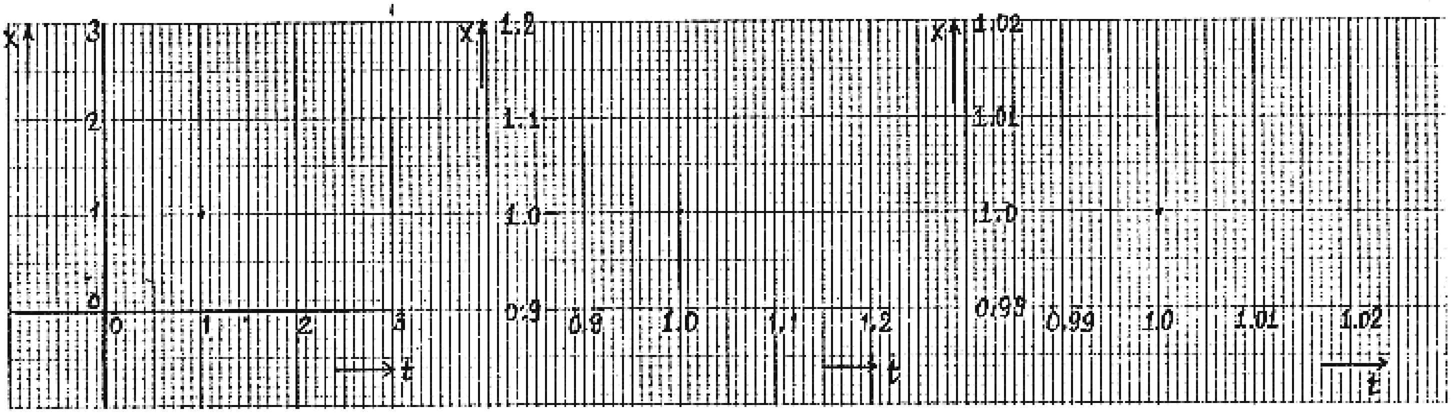
Plot the function $x = t^2$ on each of the graphs below:



Fill in this chart:

$(t+h)^3$	t^3	$3ht^2$	$3h^2t$	h^3	$= t^3 + 3ht^2 + 3h^2t + h^3$
$t = 1, h = 1$					
$t = 1, h = 0.1$					
$t = 1, h = 0.01$					

Plot the function $x = t^3$ on each of the graphs below:



- Go back to each function. On the most zoomed-in graph, calculate the slope of the "line".