NAME ______________________________ APRIL 12, 2007

MATH 145 - ELEMENTARY STATISTICS LONG EXAM III

Instructions: *Please include all relevant work to get full credit. Encircle your final answers.*

1. In a study of drying time for paints, the time to dry is found be normally distributed with $\sigma = 9$. To test the null hypothesis $H_0 : \mu = 75$, where μ denotes the mean drying time of paints in minutes, versus the alternative that $H_1 : \mu > 75$, a random sample of size $n = 25$ observations are tested.

a. Specify the appropriate test statistic to use and define your rejection rule using a level of significance of $\alpha = 0.05$. [4]

$z = \dfrac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} \sim N(0,1) \Rightarrow$ Reject H_0 when $z_{\text{obs}} > 1.645$.

b. If $\bar{x}_{\text{obs}} = 77.7$, what is your conclusion? [6]

$z_{\text{obs}} = \dfrac{77.7 - 75}{9/\sqrt{25}} = 1.5$. Since $z_{\text{obs}} = 1.5 \not> 1.645$, we do not reject H_0.

c. Compute the $p-$value if $\bar{x}_{\text{obs}} = 77.7$. [5]

$p-$value $= P(Z \geq 1.5) = 1 - 0.9332 = 0.0668$.

d. Compute the probability of committing a type I error, if you decide to reject H_0 when $\bar{x}_{\text{obs}} > 81$. [6]

$P(\bar{X} > 81) = P(Z > \frac{81-75}{9/\sqrt{25}}) = P(Z > 3.33) = 1 - 0.9996 = 0.0004$.

e. If $\bar{x}_{\text{obs}} = 77.7$, construct and interpret a 95% confidence interval for μ. [7]

$= 77.7 \pm 1.96(\frac{9}{5}) = 77.7 \pm 3.528 = [74.172, 81.228]$.

We are 95% confident that μ *is between* 74.172 *and* 81.228.

f. How large a sample should you get in order for the margin of error in part (e) to be no more than 2? [5]

$n = \left[\dfrac{1.96(9)}{2}\right]^2 \approx 77.8 \Rightarrow n = 78$.

2. A random sample of seven measurements gave $\bar{x} = 9.4$ and $s^2 = 1.94$.

a. What assumptions must you make concerning the population in order to test a hypothesis about σ^2? *Data come from a normal population.* [2]

b. Suppose the assumptions in part (a) are satisfied. Test the null hypothesis, $\sigma^2 = 0.8$, against the alternative hypothesis that $\sigma^2 > 0.8$. Use $\alpha = 0.05$. [8]

i. $H_0 : \sigma^2 = 0.8$ *vs.* $H_1 : \sigma^2 > 0.8$.

ii. $\alpha = 0.05$.

iii. *Use* $X^2 = \dfrac{(n-1)S^2}{\sigma^2} \sim \chi^2_6$.

iv. *Reject* H_0 *if* $x^2_{\text{obs}} > 12.59$.

v. $x^2_{\text{obs}} = \dfrac{(7-1)(1.94)}{0.8} = 14.55$.

vi. *Since* $x^2_{\text{obs}} = 14.55 > 12.59$, *we reject* H_0.

3. In a winter of an epidemic flu, babies were surveyed by a well-known pharmaceutical company to determine if the company's new medicine was effective after two days. Among 120 babies who had the flu and were given the medicine, 29 were cured within two days. Among 280 babies who had the flu but were not given the medicine, 56 were cured within two days.

a. Formulate the most appropriate null and alternative hypotheses. Define clearly the parameters you use in your hypotheses. [4]

p_1 = *the proportion of treated babies who are cured in 2 days, and*

p_2 = *the proportion of untreated babies who are cured in 2 days.*

$H_0 : p_1 = p_2$ *vs.* $H_1 : p_1 > p_2$.

b. Specify the most appropriate test statistic and define your rejection rule using a level of significance of $\alpha = 0.05$. [4]

$$Z = \frac{\hat{p}_1 - \hat{p}_2 - (p_1 - p_2)}{\sqrt{\hat{p}(1-\hat{p})(\frac{1}{n_1} + \frac{1}{n_2})}} \approx N(0,1) \Rightarrow \textit{Reject } H_0 \textit{ if } z_{\text{obs}} > 1.645.$$

c. Test the company's claim of the effectiveness of the medicine. Write a practical conclusion. [9]

$$Z = \frac{(\frac{29}{120} - \frac{56}{280}) - 0}{\sqrt{\frac{85}{400}(1 - \frac{85}{400})(\frac{1}{120} + \frac{1}{280})}} = \frac{0.242 - 0.2}{0.0446} = 0.94.$$

d. Compute the p-value of this sample. [5]

$p-$value $= P(Z \geq 0.94) = 1 - 0.8264 = 0.1736.$

e. Construct and interpret a 99% confidence interval for $(p_1 - p_2)$. [9]

$$= (0.242 - 0.2) \pm 2.575\sqrt{\frac{0.242(1-0.242)}{120} + \frac{0.2(1-0.2)}{280}}$$
$$= 0.042 \pm 2.575(0.0458) = 0.042 \pm 0.118 = [-0.076, 0.16].$$

We are 99% confident that $p_1 - p_2$ *is between* -0.076 *and* 0.16.

4. To test the claim that the resistance of electric wire can be reduced by alloying, 21 values obtained for alloyed wire yielded $\bar{x}_1 = 0.083$ ohm and $s_1 = 0.03$ ohm, and 21 values obtained for standard wire yielded $\bar{x}_2 = 0.136$ ohm and $s_2 = 0.02$ ohm. Assume that the true variances are equal.

a. Formulate the most appropriate null and alternative hypotheses to test the claim. Define clearly the parameters you use in your hypotheses. [5]

Let μ_1=*mean resistance of alloyed wires, and* m_2=*mean resistance of standard wires.*

$H_0 : \mu_1 - \mu_2 = 0$ vs. $H_1 : \mu_1 - \mu_2 < 0$.

b. Specify the most appropriate test statistic and define your rejection rules using $\alpha = 0.01$. [5]

$$T = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \sim t_{df=21+21-2=40} \Rightarrow \textit{Reject } H_0 \textit{ if } T_{\text{obs}} < -2.423.$$

c. Test the claim using $\alpha = 0.01$, then write a practical conclusion. [10]

$$T_{\text{obs}} = \frac{(0.083 - 0.136) - 0}{\sqrt{\frac{20(.03^2)+20(.02^2)}{40}}\sqrt{\frac{1}{21} + \frac{1}{21}}} = \frac{-0.053}{0.0255\sqrt{\frac{2}{21}}} \approx -6.73.$$

Since $T_{\text{obs}} \approx -6.73 < -2.423$, *we reject* H_0. *Therefore, the mean resistance of alloyed wires is statistically lower than the mean resistance of standard wires.*

d. Construct and interpret a 99% confidence interval for $(\mu_1 - \mu_2)$. [8]

$$= (0.083 - 0.136) \pm 2.704(0.0255)\sqrt{\frac{2}{21}} = -0.053 \pm 0.021 = [-.074, -.032].$$

We are 99% confident that $\mu_1 - \mu_2$ *is between* $-.074$ *and* $-.032$.