Conditional Probability

- **Definition.** For any two events A and B with $P(B) > 0$, the *conditional probability of A given that B has occurred* is defined by

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Example 1. In a small town of 2000 people, there are 800 males, 700 of whom are employed. If a total 250 people are unemployed in this town, find the probability that a randomly selected person is

1. a male?
2. employed?
3. an employed male?
4. employed given his a male?
5. male given the person is employed?

Example 2. Suppose that of all individuals buying a certain digital camera, 60% include an optional memory card in their purchase, 40% include an extra battery, and 30% include both a card and battery.

1. Given that the selected individual purchased an extra battery, what is the probability that an optional memory card was also purchased?
2. What is the probability that an extra battery is included in the purchase given that the person got the optional memory card?

- **The Multiplication Rule.**

$$P(A \cap B) = P(A|B) \cdot P(B) \quad \text{or} \quad P(A \cap B) = P(B|A) \cdot P(A)$$

Example 3. A chain of video stores sells three different brands of VCRs. Of its VCR sales, 50% are brand 1 (the least expensive), 30% are brand 2, and 20% are brand 3. Each manufacturer offers a 1-year warranty on parts and labor. It is known that 25% of brand 1's VCRs require warranty repair work, whereas the corresponding percentages for brands 2 and 3 are 20% and 10%, respectively.

1. What is the probability that a randomly selected purchaser has bought a brand 1 VCR that will need repair while under warranty?
2. What is the probability that a randomly selected purchaser has a VCR that will need repair while under warranty?
3. If a customer returns to the store with a VCR that needs warranty repair work, what is the probability that it is a brand 1 VCR?

Example 4. Online chat rooms are dominated by the young. Teens are the biggest users. If we look only at adult internet users (age 18 and over), 47% of the 18 to 29 age group chat, as do 21% of those aged 30 to 49 and just 7% of those 50 and over. It is known that 29% of adult internet users are age 18 to 29, another 47% are 30 to 49 years old, and the remaining 24% are age 50 and over. If an adult internet user is randomly selected,

1. what is the probability that the person is at least 50 years old?
2. what is the probability that the person is at least 50 years old and chat online?
3. what is the probability that the person is at least 50 years old given that the person chat online?
4. what is the probability that the person chat online given he/she is at least 50 years old?

- **Total Probability.** Let $A_1, \ldots, A_k$ be mutually exclusive and complementary events (That is, $A_1, \ldots, A_k$ form a partition of the sample space). Then for any other event B,

$$\begin{aligned} P(B) &= P(B \cap A_1) + P(B \cap A_2) + \cdots + P(B \cap A_k) \\ &= P(B|A_1)P(A_1) + P(B|A_2)P(A_2) + \cdots + P(B|A_k)P(A_k) \end{aligned}$$

- **Bayes' Theorem.** Let $A_1, \ldots, A_k$ be a collection of k mutually exclusive and complementary events with $P(A_i) > 0$ for $i = 1, \ldots, k$. Then for any other event B for which $P(B) > 0$,

$$P(A_j|B) = \frac{P(A_j \cap B)}{P(B)} = \frac{P(B|A_j)P(A_j)}{\sum_{i=1}^{k} P(B|A_i)P(A_i)}$$

Practice.

1. Do #3.136 on page 177

2. Do #3.166 on page 183

3. A company uses three different assembly lines $-A_1, A_2, A_3-$ to manufacture a particular component. Of those manufactured by A_1, 5% need rework to remedy a defect, whereas 8% of A_2's components need rework and 10% of A_3's need rework. Suppose that 50% of all components are produced by line A_1, 30% are produced by line A_2, and 20% come from line A_3. If a component is randomly selected, what is the probability that
   a. it needs rework?
   b. it came from line A_1 given that it requires rework?
   c. it came from line A_2 given that it requires rework?

- **Homework problems:**
Sec 3.5/3.6; (pp. 155-161) # 63, 64, 65, 66, 72, 73, 76, 82, 83, 85.
Sec 3.9; (pp. 176-177) # 129, 130, 133