Confidence Interval for a Population Mean (σ^2 is known)

1. **Important Results:**

   a. If $X \sim N(\mu, \sigma)$, then $\bar{X} \sim N(\mu, \frac{\sigma}{\sqrt{n}}) \quad \Rightarrow \quad Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1)$

   b. If $n > 30$, then $\bar{X} \approx N(\mu, \frac{\sigma}{\sqrt{n}}) \quad \Rightarrow \quad Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \approx N(0, 1)$

2. **Notation.** The value z_α is defined as the value of the standard normal random variable z such that the area α will lie to its right. In other words, $P(Z > z_\alpha) = \alpha$.

3. The (1-α)100% *Confidence Interval* for the population mean (μ) is

$$[\bar{X} - Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}, \bar{X} + Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}] \tag{1}$$

4. Practice

   a. A random sample of 90 observations produced a mean $\bar{x} = 25.9$ and a standard deviation $s = 2.7$.

   i. Find a 95% confidence interval for μ. What is the margin of error?

   ii. Find a 90% confidence interval for μ. What is the margin of error?

   iii. Find a 99% confidence interval for μ. What is the margin of error?

   b. A random sample of 100 observations from a normally distributed population possesses a mean equal to 83.2 and a standard deviation equal to 6.4.

   i. Find a 95% confidence interval for μ.

   ii. What do you mean when you say that a confidence coefficient is 0.95?

   iii. Find a 99% confidence interval for μ.

   iv. What happens to the width of a confidence interval as the value of the confidence coefficient is increased while the sample size is held fixed?

   v. Would your confidence intervals of parts (i) and (iii) be valid if the distribution of the original population was not normal? Explain.

   c. The mean and standard deviation of a random sample of n measurements are equal to 33.9 and 3.3, respectively.

   i. Find a 95% confidence interval for μ if $n = 100$.

   ii. Find a 95% confidence interval for μ if $n = 400$.

   iii. What is the effect on the width of a confidence interval of quadrupling the sample size while holding the confidence coefficient fixed?

Homework problems:
Section 7.2: pp. 329-332; # 8, 9, 10, 11, 13, 14.