Confidence Intervals for One Population

- **Estimating the population mean (μ) when σ is known.**

  1. Then, $Z = \dfrac{\bar{X} - \mu}{\sigma/\sqrt{n}}$ follows the $N(0,1)$.

  2. The $(1-\alpha)100\%$ confidence interval for μ is $\left[\bar{X} - z_{\frac{\alpha}{2}}\dfrac{\sigma}{\sqrt{n}}, \bar{X} + z_{\frac{\alpha}{2}}\dfrac{\sigma}{\sqrt{n}}\right]$.

  a. The 90% C.I. for μ is $\left[\bar{X} - 1.645\dfrac{\sigma}{\sqrt{n}}, \bar{X} + 1.645\dfrac{\sigma}{\sqrt{n}}\right]$.

  b. The 95% C.I. for μ is $\left[\bar{X} - 1.96\dfrac{\sigma}{\sqrt{n}}, \bar{X} + 1.96\dfrac{\sigma}{\sqrt{n}}\right]$.

  c. The 99% C.I. for μ is $\left[\bar{X} - 2.575\dfrac{\sigma}{\sqrt{n}}, \bar{X} + 2.575\dfrac{\sigma}{\sqrt{n}}\right]$.

  3. The *Margin of Error*, $M = Z_{\frac{\alpha}{2}}\dfrac{\sigma}{\sqrt{n}}$.

  4. For a specified margin of error M, the required sample size is $n = \left(\dfrac{Z_{\frac{\alpha}{2}}\sigma}{M}\right)^2$.

- **Estimating the population mean (μ) when σ is unknown and the sample size $n < 30$.**

  1. Then, $T = \dfrac{\bar{X} - \mu}{s/\sqrt{n}}$ follows the student t-distribution with $n-1$ degrees of freedom.

  2. The $(1-\alpha)100\%$ confidence interval for μ is $\left[\bar{X} - (t_{\frac{\alpha}{2},(n-1)})\dfrac{s}{\sqrt{n}}, \bar{X} + (t_{\frac{\alpha}{2},(n-1)})\dfrac{s}{\sqrt{n}}\right]$.

  3. The *Margin of Error*, $M = (t_{\frac{\alpha}{2},(n-1)})\dfrac{s}{\sqrt{n}}$

- **Estimating the population mean (μ) when σ is unknown but the sample size $n \geq 30$.**

  In this case, treat s as if it is σ, then use the first method. This is due to the fact that the sample size is large ($n \geq 30$) and hence, the value of s is very close to σ.

- **Estimating the population proportion (p) when the sample size is large.**

  1. The unbiased estimator of p is the sample proportion $\hat{p} = \dfrac{Y}{n}$, where Y is the number of successes in the sample.
  Recall that if $Y \sim \text{bin}(n,p)$, then $E(Y) = np$ and $Var(Y) = np(1-p)$.
  Therefore, $E(\hat{p}) = p$ and $Var(\hat{p}) = \frac{p(1-p)}{n}$.

  2. The $(1-\alpha)100\%$ confidence interval for p is $\left[\hat{p} - (z_{\frac{\alpha}{2}})\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}, \hat{p} + (z_{\frac{\alpha}{2}})\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}\right]$.

  3. The *Margin of Error*, $M = (z_{\frac{\alpha}{2}})\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}} \quad \Rightarrow \quad n \leq \dfrac{z^2_{\alpha/2}}{4M^2}$.

Homework problems:
Section 7.3: (pp. 340-342) # 28, 29, 30, 31, 32, 39, 40. (Due Thursday, 3/27)
Section 7.4: (pp. 349-350) # 45, 46, 47, 48, 49, 50, 54. (Due Monday, 3/31)
Section 7.5: (pp. 356-357) # 63, 64, 66, 68, 69, 73, 74, 78. (Due Monday, 3/31)