## Means and Variances of R.V.

- **Discrete Random Variable:** A *discrete random variable* $X$ has a finite (should be countable) number of possible values.

| $x$ | $x_1$ | $x_2$ | $x_3$ | $\cdots$ | $x_k$ |
|---|---|---|---|---|---|
| $\Pr(X = x)$ | $p_1$ | $p_2$ | $p_3$ | $\cdots$ | $p_k$ |

The probabilities $p_i$ must satisfy two requirements:

**1.** Every probability $p_i$ is a number between 0 and 1.

**2.** $p_1 + p_2 + \cdots + p_k = 1$

- The **Mean** of $X$, $\mu = x_1p_1 + x_2 + p_2 + \cdots + x_kp_k = \sum_{i=1}^{k} x_ip_i$

- The **Variance** of $X$,

$$\sigma^2 = (x_1 - \mu_X)^2p_1 + (x_2 - \mu_X)^2p_2 + \cdots + (x_k - \mu_X)^2p_k = \sum_{i=1}^{k}(x_i - \mu_X)^2p_i$$

- Example 1: Let $X$ be the number of cars that Linda is able to sell on a typical Saturday. The probability distribution for $X$ is given below

| Car sold ($x$) | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| Probability | 0.3 | 0.4 | 0.2 | 0.1 |

Find the mean and variance of $X$.

**1.** $\mu_X =$

**2.** $\text{Var}(X) =$

- Example 2: In the experiment of tossing an unfair coin, let $Y = 1$ if the result is a head and $Y = 0$ if tail. If the coin shows a head 70% of the time, construct the probability distribution table and compute the mean and variance of $Y$.

- Example 3: In the experiment of rolling a die, let $Z$ be the number of dots on top of the die. Construct the probability distribution table and compute the mean and variance of $Z$.

- **Law of Large Numbers:** Draw $n$ independent observations at random from any population with finite mean $\mu$. As the number of observations drawn increases, the sample average $\bar{x}$ of the observed values eventually approaches the mean $\mu$ of the population as closely as you wish and then stays that close.

- **Rules for Means:**

  **1.** If $X$ is a random variable and $a$ and $b$ are fixed numbers, then

  $$\mu_{a+bX} = a + b\mu_X$$

  **2.** If $X$ and $Y$ are random variables, then

  $$\mu_{X+Y} = \mu_X + \mu_Y$$

- **Example 4:** Linda is a sales associate at a large auto dealership. At her commission rate of 25% of profit on each vehicle she sells, Linda expects to earn $350 for each car sold and $400 for each truck or SUV sold. The probability distributions for the number of cars ($X$) and SUVs ($Y$) that she is able to sell on a typical Saturday are given below. If she has a fixed salary of $50 per day, compute Linda's expected earnings for each type of vehicle on such a day. What is her expected total income?

| Car sold ($x$) | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| Probability | 0.3 | 0.4 | 0.2 | 0.1 |

Linda estimates her truck or SUV sales is

| Vehicles sold ($y$) | 0 | 1 | 2 |
|---|---|---|---|
| Probability | 0.4 | 0.5 | 0.1 |

- **Rules for Variances:**

  **1.** If $X$ is a random variable and $a$ and $b$ are fixed numbers, then

  $$Var(a + bX) = b^2 Var(X)$$

  **2.** If $X$ and $Y$ are *independent* random variables, then

  $$Var(X \pm Y) = Var(X) + Var(Y)$$

- **Homework problems:**
  Section 4.3: pp. 202-204; # 31, 32, 36, 39, 41.