Summary

1. One population (μ)

a. If σ is known or $n \geq 30$, $Z = \dfrac{\bar{X} - \mu}{\sigma/\sqrt{n}}$ follows the $N(0,1)$.

i. The $(1-\alpha)100\%$ confidence interval for μ is $[\bar{X} - Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}, \bar{X} + Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}]$.

ii. To test $H_0 : \mu = \mu_0$ vs. $H_1 : \mu > \mu_0$, reject the null if $Z_{\text{obs}} = \dfrac{\bar{X}_{\text{obs}} - \mu}{\sigma/\sqrt{n}} > Z_\alpha$, or if the $p-$value$= P(Z \geq Z_{\text{obs}})$ is $\leq \alpha$.

iii. To test $H_0 : \mu = \mu_0$ vs. $H_1 : \mu < \mu_0$, reject the null if $Z_{\text{obs}} < -Z_\alpha$, or if the $p-$value$= P(Z \leq Z_{\text{obs}})$ is $\leq \alpha$.

iv. To test $H_0 : \mu = \mu_0$ vs. $H_1 : \mu \neq \mu_0$, reject the null if $|Z_{\text{obs}}| > Z_{\frac{\alpha}{2}}$, or if the $p-$value$= 2 * P(Z \geq |Z_{\text{obs}}|)$ is $\leq \alpha$.

b. If σ is unknown, $n < 30$, and the $X_i's$ come from a normal population, $t = \dfrac{\bar{X} - \mu}{s/\sqrt{n}}$ follows the $t-$distribution with $(n-1)$ degrees of freedom.

i. The $(1-\alpha)100\%$ confidence interval for μ is $[\bar{X} - t_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}}, \bar{X} + t_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}}]$.

ii. To test $H_0 : \mu = \mu_0$ vs. $H_1 : \mu > \mu_0$, reject the null if $t_{\text{obs}} = \dfrac{\bar{X}_{\text{obs}} - \mu}{s/\sqrt{n}} > t_\alpha$.

iii. To test $H_0 : \mu = \mu_0$ vs. $H_1 : \mu < \mu_0$, reject the null if $t_{\text{obs}} < -t_\alpha$.

iv. To test $H_0 : \mu = \mu_0$ vs. $H_1 : \mu \neq \mu_0$, reject the null if $|t_{\text{obs}}| > t_{\frac{\alpha}{2}}$.

2. One population (p)

a. If n is large ($np > 10$ and $n(1-p) > 10$), $Z = \dfrac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}} \approx N(0,1)$.

i. The $(1-\alpha)100\%$ confidence interval for p is $[\hat{p} - Z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}, \hat{p} + Z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}]$.

ii. To test $H_0 : p = p_0$ vs. $H_1 : p > p_0$, reject the null if $Z_{\text{obs}} = \dfrac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} > Z_\alpha$.

iii. To test $H_0 : p = p_0$ vs. $H_1 : p < p_0$, reject the null if $Z_{\text{obs}} < -Z_\alpha$.

iv. To test $H_0 : p = p_0$ vs. $H_1 : p \neq p_0$, reject the null if $|Z_{\text{obs}}| > Z_{\frac{\alpha}{2}}$.

3. One population (σ^2): When the $X_i's$ come from a normal population, $\dfrac{(n-1)S^2}{\sigma^2}$ follows the χ^2 distribution with $n-1$ degrees of freedom.

a. To test $H_0 : \sigma^2 = \sigma_0^2$ vs. $H_1 : \sigma^2 > \sigma_0^2$, reject the null if $X^2_{\text{obs}} = \dfrac{(n-1)S^2_{\text{obs}}}{\sigma_0^2} > \chi^2_\alpha$.

b. To test $H_0 : \sigma^2 = \sigma_0^2$ vs. $H_1 : \sigma^2 < \sigma_0^2$, reject the null if $X^2_{\text{obs}} = \dfrac{(n-1)S^2_{\text{obs}}}{\sigma_0^2} < \chi^2_{1-\alpha}$.

c. To test $H_0 : \sigma^2 = \sigma_0^2$ vs. $H_1 : \sigma^2 \neq \sigma_0^2$, reject the null if $X^2_{\text{obs}} > \chi^2_{\frac{\alpha}{2}}$ or if $X^2_{\text{obs}} < \chi^2_{1-\frac{\alpha}{2}}$.

4. Two populations (μ_1, μ_2)

a. If σ_1 and σ_2 are known (or n_1 and $n_2 \geq 30$), $Z = \dfrac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$ follows the $N(0,1)$.

i. The $(1-\alpha)100\%$ confidence interval for $\mu_1 - \mu_2$ is $(\bar{X}_1 - \bar{X}_2) \pm Z_{\frac{\alpha}{2}} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$.
ii. To test $H_0 : \mu_1 - \mu_2 = d_0$ vs. $H_1 : \mu_1 - \mu_2 > d_0$, reject the null if $Z_{\text{obs}} > Z_\alpha$.
iii. To test $H_0 : \mu_1 - \mu_2 = d_0$ vs. $H_1 : \mu_1 - \mu_2 < d_0$, reject the null if $Z_{\text{obs}} < -Z_\alpha$.
iv. To test $H_0 : \mu_1 - \mu_2 = d_0$ vs. $H_1 : \mu_1 - \mu_2 \neq d_0$, reject the null if $|Z_{\text{obs}}| > Z_{\frac{\alpha}{2}}$.

b. If σ_1 and σ_2 are unknown, $t = \dfrac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$ follows **approximately** $t-$distribution with k degrees of freedom, where k is approximated by

$$k \approx \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{1}{n_1-1}\left(\frac{s_1^2}{n_1}\right)^2 + \frac{1}{n_2-1}\left(\frac{s_2^2}{n_2}\right)^2}$$

i. The $(1-\alpha)100\%$ confidence interval for $\mu_1 - \mu_2$ is $(\bar{X}_1 - \bar{X}_2) \pm t_{\frac{\alpha}{2}} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$.
ii. To test $H_0 : \mu_1 - \mu_2 = d_0$ vs. $H_1 : \mu_1 - \mu_2 > d_0$, reject the null if $t_{\text{obs}} > t_\alpha$.
iii. To test $H_0 : \mu_1 - \mu_2 = d_0$ vs. $H_1 : \mu_1 - \mu_2 < d_0$, reject the null if $t_{\text{obs}} < -t_\alpha$.
iv. To test $H_0 : \mu_1 - \mu_2 = d_0$ vs. $H_1 : \mu_1 - \mu_2 \neq d_0$, reject the null if $|t_{\text{obs}}| > t_{\frac{\alpha}{2}}$.

c. If σ_1 and σ_2 are unknown **but can be assumed to be equal**, $t = \dfrac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}}}$ follows $t-$distribution with (n_1+n_2-2) degrees of freedom, where $s_p^2 = \dfrac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}$.

i. The $(1-\alpha)100\%$ confidence interval for $\mu_1 - \mu_2$ is $(\bar{X}_1 - \bar{X}_2) \pm t_{\frac{\alpha}{2}} \sqrt{\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}}$.
ii. To test $H_0 : \mu_1 - \mu_2 = d_0$ vs. $H_1 : \mu_1 - \mu_2 > d_0$, reject the null if $t_{\text{obs}} > t_\alpha$.
iii. To test $H_0 : \mu_1 - \mu_2 = d_0$ vs. $H_1 : \mu_1 - \mu_2 < d_0$, reject the null if $t_{\text{obs}} < -t_\alpha$.
iv. To test $H_0 : \mu_1 - \mu_2 = d_0$ vs. $H_1 : \mu_1 - \mu_2 \neq d_0$, reject the null if $|t_{\text{obs}}| > t_{\frac{\alpha}{2}}$.

d. For **paired** observations (the two samples are **NOT** independent), work with $d_i = x_i - y_i$.

i. To test $H_0 : \mu_D = d_0$ vs. $H_1 : \mu_D > d_0$, reject the null if $t_{\text{obs}} = \dfrac{\bar{d} - d_0}{S_D/\sqrt{n}} > t_\alpha(n-1)$.
ii. To test $H_0 : \mu_D = d_0$ vs. $H_1 : \mu_D < d_0$, reject the null if $t_{\text{obs}} < -t_\alpha(n-1)$.
iii. To test $H_0 : \mu_D = d_0$ vs. $H_1 : \mu_D \neq d_0$, reject the null if $|t_{\text{obs}}| > t_{\frac{\alpha}{2}}(n-1)$.

5. Two populations (p_1, p_2): If n_1 and n_2 are large ($n_i p_i > 10$ and $n_i(1-p_i) > 10$),

$$Z = \frac{\hat{p}_1 - \hat{p}_2 - (p_1 - p_2)}{\sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}} \approx N(0,1).$$

a. The $(1-\alpha)100\%$ confidence interval for $p_1 - p_2$ is $\left[(\hat{p}_1 - \hat{p}_2) \pm Z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}\right]$.

b. To test $H_0 : p_1 - p_2 = 0$ vs. $H_1 : p_1 - p_2 > 0$, reject the null if $Z_{\text{obs}} = \dfrac{(\hat{p}_1 - \hat{p}_2) - 0}{\sqrt{\frac{\hat{p}(1-\hat{p})}{n_1} + \frac{\hat{p}(1-\hat{p})}{n_2}}} > Z_\alpha$,

where, $\hat{p} = \dfrac{Y_1 + Y_2}{n_1 + n_2}$, the pooled sample proportion.

c. To test $H_0 : p_1 - p_2 = 0$ vs. $H_1 : p_1 - p_2 < 0$, reject the null if $Z_{\text{obs}} < -Z_\alpha$.

d. To test $H_0 : p_1 - p_2 = 0$ vs. $H_1 : p_1 - p_2 \neq 0$, reject the null if $|Z_{\text{obs}}| > Z_{\frac{\alpha}{2}}$.