Review: Functions

• Definition of a Function: A function f is a rule that assigns to each element x in a set D exactly one element, called f(x), in a set E.

- The Vertical Line Test: A curve in the *xy*-plane is the graph of a function of *x* if and only if no vertical line intersects the curve more than once.
- Even Function: If a function f satisfies f(-x) = f(x) for every number x in its domain, then f is called an *even function*.
- Odd Function: If a function f satisfies f(-x) = -f(x) for every number x in its domain, then f is called an odd function.
- Increasing Function: A function f is called *increasing* on an interval I if $f(x_1) < f(x_2)$ whenever $x_1 < x_2$ in I.
- Decreasing Function: A function f is called *decreasing* on an interval I if $f(x_1) > f(x_2)$ whenever $x_1 < x_2$ in I.

• Types of Functions

- 1. Linear
- 2. Polynomial
- **3.** Power
- 4. Rational
- 5. Algebraic
- 6. Trigonometric
- 7. Exponential
- 8. Logarithmic

• Vertical and Horizontal Shifts: Suppose c > 0. To obtain the graph of

- **1.** y = f(x) + c, shift the graph of y = f(x) a distance c units upward.
- **2.** y = f(x) c, shift the graph of y = f(x) a distance c units downward.
- **3.** y = f(x c), shift the graph of y = f(x) a distance c units to the right.
- **4.** y = f(x + c), shift the graph of y = f(x) a distance c units to the left.
- Vertical and Horizontal Stretching and Reflecting: Suppose c > 1. To obtain the graph of
 - **1.** y = cf(x), stretch the graph of y = f(x) vertically by a factor of c.
 - **2.** y = (1/c)f(x), shrink the graph of y = f(x) vertically by a factor of c.
 - **3.** y = f(cx), shrink the graph of y = f(x) horizontally by a factor of c.
 - **4.** y = f(x/c), stretch the graph of y = f(x) horizontally by a factor of c.
 - 5. y = -f(x), reflect the graph of y = f(x) about the x-axis.
 - **6.** y = f(-x), reflect the graph of y = f(x) about the y-axis.
- Combinations of Functions:
 - **1.** (f+g)(x) = f(x) + g(x)
 - **2.** (f-g)(x) = f(x) g(x)

3.
$$(fg)(x) = f(x)g(x)$$

- **4.** $(f/g)(x) = \left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$
- Composition of Functions: Given two functions f and g, the composite function $f \circ g$ (also called the composition of f and g) is defined by $(f \circ g)(x) = f(g(x))$
- One-to-one Function: A function f is called a it one-to-one function if it never takes on the same value twice: that is, $f(x_1) \neq f(x_2)$ whenever $x_1 \neq x_2$.
- Horizontal Line Test: A function is one-to-one if and only if no horizontal line intersects its graph more than once.
- Inverse Function: Let f be a one-to-one function with domain A and range B. Then its *inverse function* f^{-1} has domain B and range A and is defined by $f^{-1}(y) = x \iff f(x) = y$ for any y in B.