NAME _____ FEBRUARY 19, 2018

MTH 207 - CALCULUS I EXAM I

Instructions: *Please include all relevant work to get full credit. Write your solutions using proper notations. Encircle your final answers.*

1. Evaluate the limit, if it exists. If the limit does not exist, write ∞, $-\infty$, or **DNE**.

a. $\lim_{x\to -3} \dfrac{2x^2-18}{x^2+2x-3}$ [6]

b. $\lim_{x\to \infty} \dfrac{2x^2-18}{x^2+2x-3}$ [6]

c. $\lim_{x\to 2} \dfrac{x-\sqrt{x+2}}{x-2}$ [8]

d. $\lim_{t\to \infty} \sqrt{t^2+10t}-t$ [8]

e. $\lim_{x\to -1^-} \dfrac{|x-1|}{[\![x]\!]-1}$ [5]

2. Using the $\epsilon - \delta$ definition of the limit, prove that $\lim_{x \to 2} 10 - 3x = 4$. [10]

3. Prove the **Squeeze Theorem**. That is, if $f(x) \leq g(x) \leq h(x)$ when x is near a (except possibly at a) and $\lim_{x \to a} f(x) = \lim_{x \to a} h(x) = L$, then $\lim_{x \to a} g(x) = L$. [10]

4. Use the Intermediate-Value Theorem to show that the equation $x^3 + x = 4x^2 - 3$ has at least one solution between 1 and 2. [8]

5. Let

$$f(x) = \begin{cases} ax^2 + b & , x \leq 1 \\ 1/x & , x > 1 \end{cases}$$

Find the values of a and b so that $f(x)$ is differentiable at $x = 1$. [10]

6. Determine the derivative of the following functions (*You don't have to simplify*):

a. $y = 2\sqrt[3]{2-x}(2x+1)^4$ [7]

b. $y = \dfrac{3x+2}{(3-2x)^3}$ [7]

c. $f(t) = e^{(3t^2)}4^{3t}$ [8]

d. $f(t) = \ln\left(3t^2 \log_2 t\right)$ [8]

7. Find $\dfrac{dy}{dx}$ if $\tan^2(x+y) = \sin^{-1}(x) + \sec^{-1} y$. [10]

8. If $x^2y^2 = 2$, $y = f(x)$, show that $\dfrac{d^2y}{dx^2} = y^3$. [10]

9. If an object is thrown vertically, its height (in feet) after t seconds is $H(t) = -16t^2 + v_0t + h_0$, where v_0 is the initial velocity of the object and h_0 is its initial height from the ground. Suppose a ball in thrown vertically upward with an initial velocity of $v_0 = 32$ ft/sec from the top of a building that is 128 ft high.

a. What is the velocity of the ball 2 seconds later? [6]

b. When will the ball hit the ground? What is the velocity of the ball when it hits the ground? [8]

10. A paper cup has the shape of a cone with height 10 cm and radius 3 cm (at the top). If water is poured into the cup at a rate of 2 cm^3/sec, how fast is the water level rising when the water is 5 cm deep? [Note: Volume of a cone is $V = \frac{1}{3}\pi r^2 h$] [10]

11. Find the linearization $L(x)$ of $f(x) = \sqrt[3]{1+6x}$ at $a = 0$. Then use it to estimate the value of $\sqrt[3]{1.005}$ [10]