Name ______________________________ August 10, 2018

MTH 207 - Calculus I Final Exam

Instructions: *Include all relevant work to get full credit. Write your solutions using proper notations. Encircle your final answers.*

1. Evaluate the limit, if it exists. If the limit does not exist, write ∞, $-\infty$, or **DNE**.

a. $\lim\limits_{x\to 2} \dfrac{x^2-3x+2}{4x-2x^2}$ [8]

b. $\lim\limits_{x\to 0} \dfrac{\sqrt{x+4}-2}{x}$ [8]

c. $\lim\limits_{x\to\infty} \sqrt{x^2+4t}-t$ [10]

2. Let $h(x) = f(x)+g(x)$. Prove that $h'(x) = f'(x)+g'(x)$ using the limit definition of the derivative, assuming both $f'(x)$ and $g'(x)$ exist. [10]

3. Determine the derivative of the following functions (*You don't have to simplify*):

a. $y = (2x+1)^3 e^{(3x^2)}$ [8]

b. $f(\theta) = \left(\dfrac{e^{4\theta}}{\sin^2(3\theta)}\right)^5$ [8]

c. $f(x) = \ln\left(\dfrac{3x^2(2-x)^3}{(x^2-1)^2}\right)$ [10]

4. If $x^2y^2 = 2$, $y = f(x)$, show that $\dfrac{d^2y}{dx^2} = y^3$. [12]

5. Consider the function, $f(x) = x(x-4)^3$.

a. Construct the first derivative chart that shows the first-order critical values and where $f'(x)$ is positive and negative. Also, include in this chart where the $f(x)$ is increasing and decreasing. Then specify if there is any local maximum and/or local minimum points. [10]

b. Construct the second derivative chart that shows the second-order critical value and where $f''(x)$ is positive and negative. Also, include in this chart where the $f(x)$ is concave up and concave down. Then specify if there is any inflection points. [8]

c. Sketch the graph of $f(x)$ showing the relevant points and the results of parts (a) and (b). [6]

6. A paper cup has the shape of a cone with height 10 cm and radius 6 cm (at the top). If water is poured into the cup at a rate of 2 cm^3/sec, how fast is the water level rising when the water is 5 cm deep? [Note: Volume of a cone is $V = \frac{1}{3}\pi r^2 h$] [12]

7. If 1200 cm^2 of material is available to make a box with a square base and an open top, find the dimensions of the box with the largest possible volume. What is the largest possible volume? *Make sure that you check/verify that you get a maximum volume with this dimensions.* [15]

8. Use the **Fundamental Theorem of Calculus - Part I** to prove the **Fundamental Theorem of Calculus - Part II**. That is, if $f(x)$ is continuous on $[a, b]$, then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

where $F(x)$ is any antiderivative of $f(x)$. [10]

9. Evaluate the following integrals:

a. $\int (8x^3 - 10e^{5x} + \sqrt{x} - x^{-1})\,dx$ [10]

b. $\displaystyle\int_1^4 \frac{10+5x^2}{2\sqrt{x}}\,dx$ [10]

c. $\displaystyle\int_0^{\pi/4} (1+\tan\theta)^3 \sec^2\theta\, d\theta$ [10]

10. A stone is thrown upward with an initial velocity of 64 ft/sec from the edge of a cliff 336 ft above the ground. If the only force acting on this ball is Earth's gravity ($a = -32$ ft/sec^2), answer the following questions:

a. Determine the velocity of the ball, $v(t)$, t seconds later. What is the velocity of the ball 1 second later? When will the ball attain its maximum height? [10]

b. Determine the height function, $h(t)$, t seconds later. When will the ball hit the ground and at what velocity will it hit the ground? [10]

11. Calculate the volume of the solid of revolution when the region enclosed by the curves $y = x$ and $y = 5x - x^2$ is rotated about

a. the $x-$axis using the disk/washer method. [10]

b. the $y-$axis using the cylindrical shell method. [10]