List of Proofs for the Final Exam

1. Prove that $\lim_{x\to a}[f(x)+g(x)] = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$, using the $\epsilon - \delta$ definition, assuming both limits exist.

2. Prove the **Squeeze Theorem** using the $\epsilon - \delta$ definition.

3. Prove that $D_x[f(x)+g(x)] = f'(x)+g'(x)$ using the limit definition of the derivative, assuming both $f'(x)$ and $g'(x)$ exist.

4. State and prove the **Rolle's Theorem**.

5. Use the **Rolle's Theorem** to prove the **Mean Value Theorem**.

6. Prove

- **a. Theorem 4.2.5:** If $f'(x) = 0$ for all x in an interval (a, b), then $f(x)$ is constant on (a, b).
- **b. Corollary 4.2.7:** If $f'(x) = g'(x)$ for all x in an interval (a, b), then $f - g$ is constant on (a, b); that is, $f(x) = g(x) + c$ were c is a constant.

7. Use the **Fundamental Theorem of Calculus - Part I** to prove the **Fundamental Theorem of Calculus - Part II**. That is, if $f(x)$ is continuous on $[a, b]$, then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

where $F(x)$ is any antiderivative of $f(x)$.

I will choose one problem from 1 - 3 and another problem from 4 - 7 to be included in the final exam.