

Instructions: Include all relevant work to get full credit.

Homework 1 - Algebra Review (20-points)

1. Evaluate each expression without using a calculator.

a. $(-2)^3 = (-2)(-2)(-2) = -8$

b. $-2^3 = -(2)(2)(2) = -8$

c. $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

d. $\frac{3^{25}}{3^{21}} = 3^4 = 81$

e. $\left(\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$

f. $16^{-3/4} = \frac{1}{16^{3/4}} = \frac{1}{(\sqrt[4]{16})^3} = \frac{1}{2^3} = \frac{1}{8}$

2. Simplify each expression. Write your answer without negative exponents.

a. $\sqrt{300} - \sqrt{48} = \sqrt{100}\sqrt{3} - \sqrt{16}\sqrt{3} = \sqrt{3}(10-4) = 6\sqrt{3}$

b. $(3a^3b^4)(2a^2b)^2 = (3a^3b^4)(4a^4b^2) = 12a^7b^6$

c. $\left(\frac{3x^{3/2}y^3}{x^2y^{-1/2}}\right)^2 = \frac{3^2x^3y^6}{x^4y^{-1}} = \frac{9y^7}{x}$

3. Expand and simplify.

a. $3(x+6) + 4(2x-5) = 3x+18+8x-20 = 11x-2$

b. $(x+3)(3x-2) = 3x^2+7x-6$

c. $(\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b}) = a-b$

d. $(2x-3)^2 = 4x^2-12x+9$

e. $(x-2)^3 = (x-2)(x^2-4x+4) = x^3-6x^2+12x-8$

4. Factor each expression.

a. $9x^2-25 = (3x-5)(3x+5)$

b. $2x^2+5x-12 = (2x-3)(x+4)$

c. $(x^3-3x^2)-4x+12 = x(x^2-4)-3(x^2-4) = (x-3)(x^2-4) = (x-3)(x-2)(x+2)$

d. $x^4+27x = x(x^3+27) = x(x+3)(x^2-3x+9)$

e. $3x^{3/2}-9x^{1/2}+6x^{-1/2} = x^{-1/2}(3x^2-9x+6) = \frac{3(x^2-3x+2)}{\sqrt{x}} = \frac{3\sqrt{x}(x-2)(x-1)}{x}$

f. $x^3y-4xy = xy(x^2-4) = xy(x-2)(x+2)$

5. Simplify the rational expression.

a. $\frac{x^2+3x+2}{x^2-x-2} = \frac{(x+2)(x+1)}{(x-2)(x+1)} = \frac{x+2}{x-2}$

b. $\frac{2x^2-x-1}{x^2-9} \cdot \frac{x+3}{2x+1} = \frac{(2x+1)(x-1)}{(x-3)(x+3)} \cdot \frac{(x+3)}{(2x+1)} = \frac{x-1}{x-3}$

c. $\frac{x^2}{x^2-4} - \frac{x+1}{x+2} = \frac{x^2-(x+1)(x-2)}{(x-2)(x+2)} = \frac{x^2-(x^2-x-2)}{(x-2)(x+2)} = \frac{x+2}{(x-2)(x+2)} = \frac{1}{x-2}$

d. $\frac{\frac{y}{x} - \frac{x}{y}}{\frac{1}{y} - \frac{1}{x}} = \frac{\frac{y^2-x^2}{xy}}{\frac{x-y}{xy}} = \frac{(y-x)(y+x)}{xy} \cdot \frac{xy}{(x-y)} = -(y+x)$

6. Rationalize the expression and simplify.

a. $\frac{\sqrt{10}}{\sqrt{5}-2} \cdot \frac{\sqrt{5}+2}{\sqrt{5}+2} = \frac{\sqrt{50} + 2\sqrt{10}}{5-4} = 5\sqrt{2} + 2\sqrt{10}$

b. $\frac{\sqrt{4+h}-2}{h} \cdot \frac{\sqrt{4+h}+2}{\sqrt{4+h}+2} = \frac{5-4}{(4+h)-4} = \frac{1}{h(\sqrt{4+h}+2)} = \frac{1}{\sqrt{4+h}+2}$

7. Solve the equation. Find only the real solutions.

a. $x+5=14-\frac{1}{2}x \Rightarrow \frac{3}{2}x=9 \Rightarrow x=6$

b. $\frac{2x}{x+1} = \frac{2x-1}{x} \Rightarrow 2x^2 = (2x-1)(x+1) \Rightarrow 2x^2 = 2x^2 + x - 1 \rightarrow x=1$

c. $x^2+x-12=0 \Rightarrow (x-3)(x+4)=0 \Rightarrow x=3 \text{ or } x=-4$

d. $2x^2+4x+1=0 \Rightarrow x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-4 \pm \sqrt{16-4(2)(1)}}{2(2)} = \frac{-4 \pm \sqrt{8}}{4} = -1 \pm \frac{1}{2}\sqrt{2}$

e. $x^4-3x^2+2=0 \Rightarrow (x^2-2)(x^2-1)=0 \Rightarrow x^2=2 \text{ or } x^2=1 \Rightarrow x=\pm 1, \pm \sqrt{2}$

f. $4|x-3|=10 \Rightarrow |x-3|=\frac{10}{4} \Rightarrow x-3=\pm \frac{5}{2} \Rightarrow x=3 \pm \frac{5}{2} = \{\frac{1}{2}, \frac{11}{2}\}$

g. $2x(4-x)^{-1/2} - 3\sqrt{4-x} = 0$

$\Rightarrow \frac{2x}{\sqrt{4-x}} - 3\sqrt{4-x} = 0 \Rightarrow \frac{2x-3(4-x)}{\sqrt{4-x}} = 0 \Rightarrow \frac{5x-12}{\sqrt{4-x}} = 0 \Rightarrow 5x-12=0 \Rightarrow x=\frac{12}{5}$

8. Solve each inequality. Write your answer using interval notation.

a. $-7 < 5-3x \leq 20 \Rightarrow -12 < -3x \leq 15 \Rightarrow 4 > x \geq -5 \Rightarrow \{x | -5 \leq x < 4\} = [-5, 4)$

b. $x^2 < x+12 \Rightarrow x^2-x-12 < 0 \Rightarrow (x-4)(x+3) < 0$

c. $x(x-1)(x+2) > 0$

$\begin{array}{c|ccc|ccc|ccc} x & - & - & - & + & + & + & + & + & + \\ x-1 & - & - & - & - & - & - & - & - & - \\ x+2 & - & - & - & + & + & + & + & + & + \\ \hline \text{Prod.} & - & - & - & + & + & + & + & + & + \end{array}$

d. $|x-5| < 4$

$\Rightarrow -4 < x-5 < 4 \Rightarrow +1 < x < 9 \Rightarrow \text{Ans: } (1, 9)$

e. $\frac{2x-3}{x+1} \leq 1 \Rightarrow \frac{2x-3}{x+1} - 1 \leq 0 \Rightarrow \frac{(2x-3)-(x+1)}{x+1} \leq 0 \Rightarrow \frac{x-4}{x+1} \leq 0$

$\begin{array}{c|ccc|ccc} x-4 & - & - & - & + & + & + & + \\ x+1 & - & - & - & + & + & + & + \\ \hline \text{Q} & + & - & - & - & + & + & + \end{array} \Rightarrow \text{Ans: } (-1, 4]$

9. State whether each equation is true or false.

a. $(p+q)^2 = p^2 + q^2 \rightarrow \text{False}$

b. $\sqrt{ab} = \sqrt{a}\sqrt{b} \rightarrow \text{True}$

c. $\sqrt{a^2+b^2} = a+b \rightarrow \text{False}$

d. $\frac{1+TC}{C} = 1+T \rightarrow \text{False}$

e. $\frac{1}{x-y} = \frac{1}{x} - \frac{1}{y} \rightarrow \text{False}$

f. $\frac{1/x}{a/x-b/x} = \frac{1}{a-b} \rightarrow \text{True}$