Homework:

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Excellent work! $\frac{30}{30}$

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#21.) $F = (d/dt)(mv)$

$F = ma$, where $a = \frac{dv}{dt}$

$m = m_0/\sqrt{1-v^2/c^2}$

m_0 = Mass of particle at rest

c = Speed of Light (CONSTANT)

$$F = \frac{m_0 a}{(1-v^2/c^2)^{3/2}}$$

PROOF $\Rightarrow$ $$F = \frac{d(mv)}{dt} = \frac{d}{dt}\left[\frac{m_0 v}{\sqrt{1-v^2/c^2}}\right] = \frac{d}{dt}\left[\frac{m_0 v}{\sqrt{\frac{c^2-v^2}{c^2}}}\right]\cdot\frac{\sqrt{c^2}}{\sqrt{c^2}}$$

$$\Rightarrow \frac{d}{dt}\left[\frac{m_0 v c}{\sqrt{c^2-v^2}}\right] = m_0 c\left[\frac{(1)\frac{dv}{dt}\cdot\sqrt{c^2-v^2} - (v)(\frac{1}{2})(c^2-v^2)^{-1/2}(-2v)\frac{dv}{dt}}{(\sqrt{c^2-v^2})^2}\right]$$

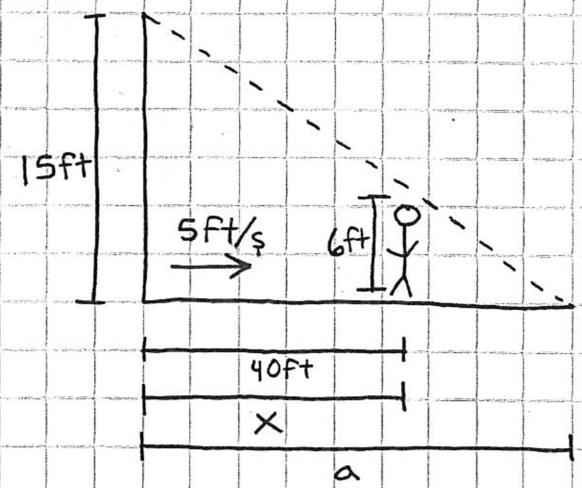
$$\Rightarrow m_0 c\left[\frac{\frac{dv}{dt}(c^2-v^2) + \frac{dv}{dt}(v^2)}{(c^2-v^2)^{1/2}(c^2-v^2)^1}\right] = m_0 c\,\frac{(c^2)\frac{dv}{dt} - (v^2)\frac{dv}{dt} + (v^2)\frac{dv}{dt}}{(c^2-v^2)^{3/2}}$$

$$= m_0 c\,\frac{(c^2)\frac{dv}{dt}}{(c^2-v^2)^{3/2}} = m_0 \frac{c^3\frac{dv}{dt}/c^3}{(c^2-v^2)^{3/2}/c^3}$$

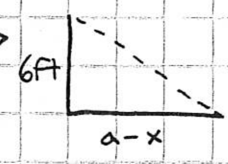
$$= m_0 \frac{dv/dt}{\left(1-\frac{v^2}{c^2}\right)^{3/2}}$$

$\bigstar$ - If $a = \frac{dv}{dt}$, $F = \frac{m_0 a}{(1-v^2/c^2)^{3/2}}$

#15.)



$\Rightarrow$



$*\quad \frac{a-x}{a} = \frac{6}{15} \Rightarrow -\frac{2}{5}a + a = x$

$\Rightarrow \frac{5}{5}a - \frac{2}{5}a = x$

$\Rightarrow \underline{\underline{x = \tfrac{3}{5}a}}$

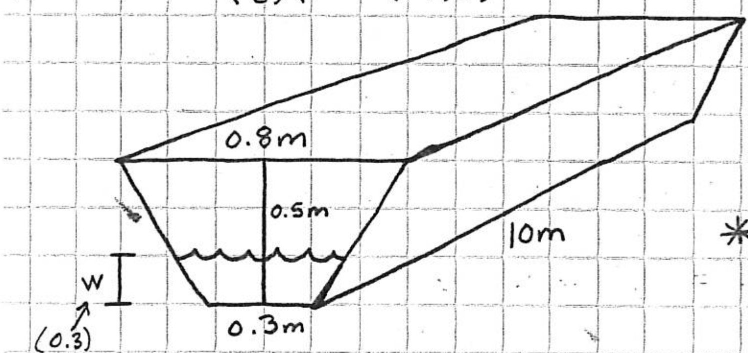
$\frac{dx}{dt} = \frac{3}{5}\cdot\frac{da}{dt}$

$\hookrightarrow \frac{da}{dt} = \frac{5}{3}\cdot\frac{dx}{dt}$

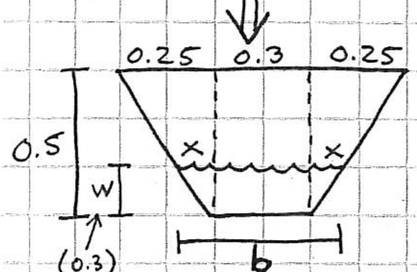
$= \left(\frac{5}{3}\right)(5) = \boxed{\frac{25}{3}}$

☆ - The tip of the man's shadow when he is 40ft from the pole is moving $\frac{25}{3}$ ft/s.

#27.) $V = \left(\frac{1}{2}\right)(a+b)(h)(L)$



$\Downarrow$



$\underline{\text{Rate}} = 0.2\,m^3/min$

$\hookrightarrow \frac{dV}{dt} = 0.2\,m^3/min$

$*\quad 0.8 - 0.3 = 0.5 \Rightarrow \frac{0.5}{2} = \underline{\underline{0.25}}$

$\hookrightarrow \frac{w}{x} = \frac{0.5}{0.25} = 2$

$\Rightarrow \underline{\underline{w = 2x}}$

$b = 0.3 + x + x$

$= 0.3 + 2x$

$= 0.3 + w$

$*\quad V = \frac{1}{2}w(0.3 + b)(10)$

$\hookrightarrow V = \frac{1}{2}w(0.3 + (0.3 + w))(10)$

$= \frac{1}{2}w(0.6 + w)(10)$

$= \underline{\underline{3w + 5w^2}}$

$\Rightarrow \frac{dV}{dt} = (3)\frac{dw}{dt} + (10w)\frac{dw}{dt}$

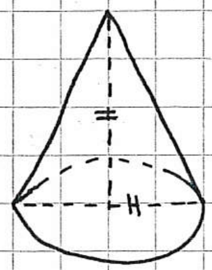
$\Rightarrow (0.2) = \frac{dw}{dt}\left(3 + 10(0.3)\right)$

$\Rightarrow 0.2/(3+3) = \frac{dw}{dt}$

$\Rightarrow \frac{dw}{dt} = \boxed{\frac{1}{30}}$

☆ - The water level in the trough is rising $1/30$ m/min

#29.)



$\underline{Rate} = 30\,ft^3/min$

$\hookrightarrow \frac{dv}{dt} = 30\,ft^3/min$

* Diameter = Height

$\Rightarrow V = \frac{1}{3}\pi r^2 h$

$\Rightarrow V = \frac{1}{3}\pi \left(\frac{h}{2}\right)^2 h$

$\Rightarrow V = \frac{1}{3}\pi \frac{h^2}{4}\cdot h$

$\Rightarrow V = \frac{\pi}{12}\cdot h^3$

$\hookrightarrow \frac{dv}{dt} = \left(\frac{\pi}{\cancel{12}_{4}}\right)(\cancel{3}h^2) + \cancel{(h^3)(0)}$

$\frac{dv}{dt} = \frac{\pi}{4}\cdot h^2 \cdot \frac{dh}{dt}$

$\frac{(4)(30)}{\pi h^2} = \frac{120}{\pi h^2} = \frac{120}{\pi (10)^2}$

$\Longrightarrow \frac{dh}{dt} = \frac{120}{\pi 100} = \boxed{\frac{6}{5\pi}}$

☆ – The height of the pile of gravel is increasing $\frac{6}{5\pi}$ ft/min

#37.) BOYLE'S LAW: $PV = C$

$\hookrightarrow$ C is Constant

$\Rightarrow \frac{d}{dt}(PV) = \frac{d}{dt}(c)$

$\Rightarrow (P)(1)\left(\frac{dV}{dt}\right) + (V)(1)\left(\frac{dP}{dt}\right) = 0$

$\Rightarrow P\cdot\frac{dV}{dt} + V\cdot\frac{dP}{dt} = 0$

$V = 600\,cm^3$

$P = 150\,kPa$

$\frac{dP}{dt} = 20\,kPa/min$

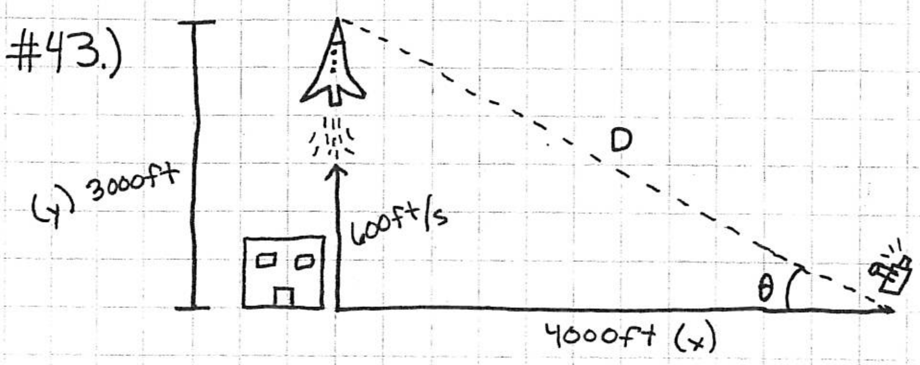
$\hookrightarrow (150)\left(\frac{dV}{dt}\right) + (600)(20) = 0$

$(150)\left(\frac{dV}{dt}\right) + 12{,}000 = 0$

$\frac{(150)\left(\frac{dV}{dt}\right)}{150} = \frac{-12{,}000}{150} \Longrightarrow \frac{dV}{dt} = \frac{-12{,}000}{150} = \boxed{-80}$

☆ – The volume of the sample of gas is decreasing at a rate of $80\,cm^3/min$

#43.)



$$D = \sqrt{x^2 + y^2}$$
$$= \sqrt{(4000)^2 + (3000)^2}$$
$$= \sqrt{25{,}000{,}000} = \underline{5000\text{ft}}$$

a.) $\Rightarrow \frac{d}{dt}(D^2) = \frac{d}{dt}(x^2 + y^2) \Rightarrow 2D \cdot \frac{dD}{dt} = 2x\left(\frac{dx}{dt}\right) + 2y\left(\frac{dy}{dt}\right)$

$$\hookrightarrow 2(5000) \cdot \frac{dD}{dt} = 2(3000)(600)$$

$$\frac{dD}{dt} = \frac{3{,}600{,}000}{10{,}000} = \boxed{360}$$

☆ - The distance from the television camera to the rocket is increasing by 360 ft/s

b.) $\tan\theta = \frac{opp}{adj} \Rightarrow \tan\theta = \frac{y}{4000} \Rightarrow \frac{d}{dt}(\tan\theta) = \frac{d}{dt}\left(\frac{y}{4000}\right)$

$$\hookrightarrow \sec^2\theta \cdot \frac{d\theta}{dt} = (y)(0) + \left(\frac{1}{4000}\right)(1) \cdot \frac{dy}{dt}$$

$$\sec^2\theta \cdot \frac{d\theta}{dt} = \frac{1}{4000} \cdot \frac{dy}{dt}$$

$$\hookrightarrow \sec^2\theta = \left(\frac{Hyp}{Adj}\right)^2 = \left(\frac{5000}{4000}\right)^2 = \left(\frac{5}{4}\right)^2 = \underline{\frac{25}{16}}$$

$$\Rightarrow \left(\frac{25}{16}\right) \cdot \frac{d\theta}{dt} = \left(\frac{1}{4000}\right)(600)$$

$$\Rightarrow \frac{d\theta}{dt} = \left(\frac{600}{4000}\right)\left(\frac{16}{25}\right) = \frac{9{,}600}{100{,}000} = \frac{96}{1000} = \boxed{\frac{12}{125}}$$

☆ - The camera's angle of elevation is changing at a rate of $^{12}/_{125}$ rad/s.