*Homework:

Colin Griffin $\frac{39.5}{40}$

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#4.) $x + y = 16$ $\hookrightarrow y = 16 - x$ | $x^2 + y^2 = b$ $\hookrightarrow b = x^2 + (16-x)^2$

$$\Rightarrow b = x^2 + (16-x)(16-x)$$
$$= x^2 + 256 - 16x - 16x + x^2$$
$$= 2x^2 - 32x + 256$$

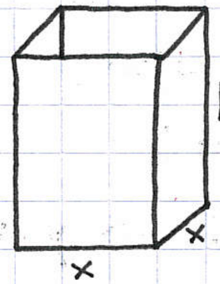
$$\Rightarrow b' = 4x - 32 = 0$$
$$+32 \quad +32$$
$$\frac{4x}{4} = \frac{32}{4} \longrightarrow \underline{\underline{x = 8}}$$ ✓

$$\Rightarrow b'' = 4 - 0 = 4 \Big\} \; \underline{\underline{b'' = 4 > 0}}$$

Thus, there's a Minimum Value at x = 8

$$\Rightarrow b = (8)^2 + (16-(8))^2$$
$$= 64 + (8)^2$$
$$= 64 + 64 = \boxed{128}$$ ✓

#15.)



$$V = x^2 \cdot h$$
$$4xh + x^2 = 1200$$
$$-x^2 \quad -x^2$$
$$\frac{4xh}{4x} = \frac{1200 - x^2}{4x} \longrightarrow h = \frac{1200 - x^2}{4x}$$ ✓

$$\Rightarrow V = x^2\left(\frac{1200 - x^2}{4x}\right) = \frac{1200x^2 - x^4}{4x} = 300x - \frac{x^3}{4}$$ ✓

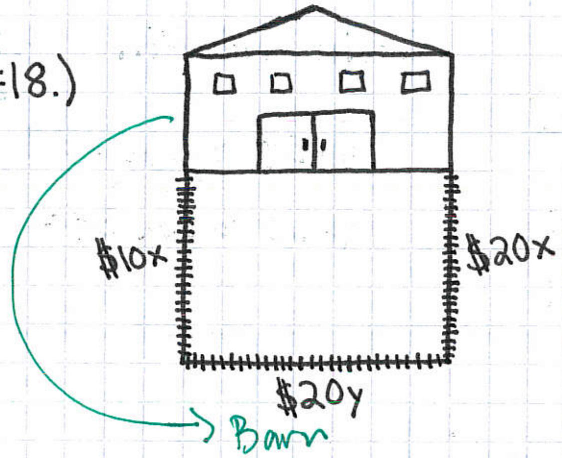
$$\Rightarrow V' = 300 - \left(\frac{1}{4}\right)(3x^2) = 0$$
$$-300 \qquad -300$$
$$-\frac{4}{3} \cdot -\frac{3}{4}x^2 = -300 \cdot -\frac{4}{3}$$
$$x^2 = \frac{1200}{3} = 400$$
$$x = \pm\sqrt{400} = \cancel{-20}, \underline{\underline{20}}$$

$$\Rightarrow V = (20)^2 \cdot \left(\frac{1200 - (20)^2}{4(20)}\right)$$
$$= 400 \cdot \left(\frac{800}{80}\right)$$
$$= 400 \cdot 10$$ ✓
$$= \boxed{4000 \text{ cm}^3}$$

$$\Rightarrow V'' = -(2)\left(\frac{3}{4}\right)x$$
$$= -\frac{6}{4}x = -\frac{3}{2}x \longrightarrow -\frac{3}{2}(20) = \underline{\underline{-30 < 0}}$$ ✓

Thus, there is a Maximum Value at x = 20

#18.)



$$A = xy$$

$$5000 = 20x + 20y + 10x$$

$$\hookrightarrow 30x + 20y = 5000$$

$$\Rightarrow y = \frac{5000 - 30x}{20} = 250 - \frac{3}{2}x \checkmark$$

$$\Rightarrow A = x\left(250 - \frac{3}{2}x\right) = 250x - \frac{3}{2}x^2 \checkmark$$

$$\Rightarrow A' = 250 - 3x = 0$$

$$\quad -250 \qquad\qquad -250$$

$$\frac{-3x}{-3} = \frac{-250}{-3} \longrightarrow \underline{\underline{x = \frac{250}{3}}} \checkmark$$

$$\Rightarrow A'' = \underline{\underline{-3 < 0}}$$

$\hookrightarrow$ Thus, there is a Maximum at $x = 250/3$.

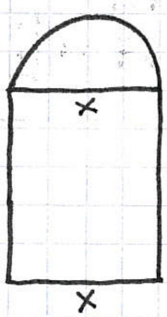
$$\Rightarrow y = 250 - \frac{\cancel{3}}{2}\left(\frac{250}{\cancel{3}}\right)$$

$$= 250 - \left(\frac{250}{2}\right)$$

$$= 250 - 125 = \underline{\underline{125}}$$

$$☆ - \boxed{x = \frac{250}{3} ft., \ y = 125 ft.}$$

#34.)



$$\text{AREA} = xy + \frac{1}{2}\pi\left(\frac{x}{2}\right)^2$$

*Note: Diameter = x

$\hookrightarrow$ Radius = x/2

$$\hookrightarrow A = xy + \frac{\pi}{8}x^2$$

$$\text{PERIMETER} = y + y + x + \left(\frac{x}{2}\right)\pi = 30 \checkmark$$

$$\Rightarrow 2y + x + \frac{x\pi}{2} - 30 = 0$$

$$-2y = \frac{2x + \pi x - 60}{2} \longrightarrow y = \frac{-2x - \pi x + 60}{4}$$

$$\Rightarrow A = x\left(\frac{-2x - \pi x + 60}{4}\right) + \frac{\pi}{8}x^2$$

$$= \frac{-2x^2 - \pi x^2 + 60x}{4} + \frac{\pi x^2}{8}$$

$$= \frac{-4x^2 - 2\pi x^2 + 120x + \pi x^2}{8}$$

$$= \frac{-4x^2 - \pi x^2 + 120x}{8} \checkmark$$

$$\xrightarrow{\text{PROOF}} : A'' = \frac{(-8-2\pi)(8)}{64} = \underline{\underline{\frac{-4-\pi}{8} < 0}} \checkmark$$

$\hookrightarrow$ Thus, there is a Maximum at $x = \frac{60}{4+\pi}$.

$$-\frac{1}{4} \quad ☆ - \boxed{x = \frac{60}{4+\pi}, \ y = \frac{\cancel{30\pi} + 30}{4+\pi}}$$

$$\Rightarrow A' = \left[\frac{(-8x - 2\pi x + 120)\cancel{(8)} - 0}{\cancel{64}\,8}\right]$$

$$A' = \frac{-8x - 2\pi x + 120}{8} = 0$$

$$-120 = -8x - 2\pi x$$

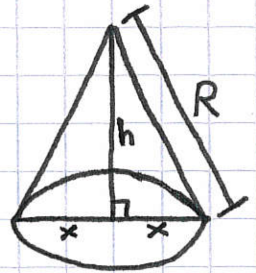
$$\frac{-120}{-2(4+\pi)} = \frac{-2(4+\pi)\cdot x}{-2(4+\pi)}$$

$$\hookrightarrow x = \frac{60}{4+\pi} \checkmark$$

$$\Rightarrow y = \frac{-2\left(\frac{60}{4+\pi}\right) - \pi\left(\frac{60}{4+\pi}\right) + 60}{4}$$

$$= \frac{\cancel{120\pi} + 120}{4(4+\pi)} \longrightarrow y = \frac{\cancel{30\pi} + 30}{4+\pi}$$

#41.)



$$h^2 + x^2 = R^2 \xrightarrow{OR} x^2 = R^2 - h^2$$

$$\hookrightarrow h = \sqrt{R^2 - x^2}$$

$$V = \frac{1}{3}\pi x^2 \cdot h$$

$$\hookrightarrow V = \frac{\pi}{3}x^2 \cdot \sqrt{R^2 - x^2}$$

$$\Rightarrow V = \frac{1}{3}\pi(R^2 - h^2)h$$

$$= \frac{1}{3}\pi R^2 h - \frac{1}{3}\pi h^3$$

$$\Rightarrow \frac{dV}{dh} = \frac{1}{3}\pi R^2 - \pi h^2 = 0$$

$$\Rightarrow h = \frac{R}{\sqrt{3}}$$

$$\Rightarrow x^2 = R^2 - \frac{R^2}{3}$$

$$= 2R^2/3$$

$$\Rightarrow V = \frac{1}{3}\pi\left(\frac{2R^2}{3}\right)\left(\frac{R}{\sqrt{3}}\right)$$

$$= \frac{2\pi R^3}{9\sqrt{3}}$$

$$\Rightarrow V' = \left(\frac{(2)\pi}{3}x\right)\left(\sqrt{R^2 - x^2}\right) + \left(\frac{\pi}{3}x^2\right)\left(\frac{1}{2}\right)\left(R^2 - x^2\right)^{-1/2}\left(-2x\right)$$

$$= \frac{2\pi x\sqrt{R^2 - x^2}}{3} - \frac{\pi x^3}{3\sqrt{R^2 - x^2}} = \frac{2\pi x(R^2 - x^2) - \pi x^3}{3\sqrt{R^2 - x^2}}$$

$$= \frac{2\pi x R^2 - 2\pi x^3 - \pi x^3}{3\sqrt{R^2 - x^2}} = \frac{2\pi x R^2 - 3\pi x^3}{3\sqrt{R^2 - x^2}} = 0$$

$$\hookrightarrow \frac{1}{3}\pi x\left(\frac{2R^2 - 3x^2}{\sqrt{R^2 - x^2}}\right) = 0$$

check: $\frac{d^2V}{dh^2} = -2\pi h < 0$

$\Rightarrow$ we get a max at $h = R/\sqrt{3}$ of $V = \frac{2\pi R^3}{9\sqrt{3}}$

$$\Rightarrow \frac{1}{3}\pi x = 0 \longrightarrow x = \cancel{0}$$

$$\Rightarrow 2R^2 - 3x^2 = 0 \longrightarrow x = \sqrt{\frac{2}{3}}R$$

$$\Rightarrow V'' = \left(\frac{1}{3}\pi\right)\left[\frac{(2R^2 - 9x^2)(\sqrt{R^2 - x^2}) - (\frac{1}{2})(R^2 - x^2)^{-1/2}(-2x)(2R^2x - 3x^3)}{(\sqrt{R^2 - x^2})^2}\right]$$

$$= \left(\frac{1}{3}\pi\right)\left[\frac{\left(\frac{(2R^2 - 9x^2)(\sqrt{R^2 - x^2})^2 + x(2R^2x - 3x^3)}{(\sqrt{R^2 - x^2})}\right)}{(\sqrt{R^2 - x^2})^2}\right] = \left(\frac{1}{3}\pi\right)\left[\frac{2R^4 - 2R^2x^2 - 9R^2x^2 + 9x^4 + 2R^2x^2 - 3x^4}{(\sqrt{R^2 - x^2})^3}\right]$$

$$= \left(\frac{1}{3}\pi\right)\left[\frac{2R^4 - 9R^2x^2 + 6x^4}{(R^2 - x^2)(\sqrt{R^2 - x^2})}\right] \xrightarrow[***]{V''(\sqrt{\frac{2}{3}}R)} = \left(\frac{1}{3}\pi\right)\left[\frac{2R^4 - 9R^2(\sqrt{\frac{2}{3}}R)^2 + 6(\sqrt{\frac{2}{3}}R)^4}{(R^2 - (\sqrt{\frac{2}{3}}R)^2)(\sqrt{R^2 - (\sqrt{\frac{2}{3}}R)^2})}\right]$$

$$= \left(\frac{1}{3}\pi\right)\left[\frac{2R^4 - 6R^4 + 8/3 R^4}{(R^2/3)(\sqrt{R^2/3})}\right] = \left(\frac{1}{3}\pi\right)\left[\frac{-4R^4 + 8/3 R^4}{(R^2/3)(\sqrt{R^2/3})}\right] = \left(\frac{1}{3}\pi\right)\left[\frac{\left(\frac{-12R^4 + 8R^4}{3}\right)}{(R^2/3)(\sqrt{R^2/3})}\right]$$

$$= \left(\frac{1}{3}\pi\right)\left[\frac{-\frac{4}{3}R^4}{\left(\frac{R^2}{3}\right)\left(\frac{R}{\sqrt{3}}\right)}\right] = \left(\frac{1}{3}\pi\right)\left(-\frac{4\cancel{R^4}^{R}}{\cancel{3}_1}\right)\left(\frac{\cancel{3}^1\sqrt{3}}{\cancel{R^3}_1}\right) = \left(\frac{1}{3}\pi\right)\left(\frac{-4R\cdot\sqrt{3}}{1}\right)$$

$$= \frac{-4R\cdot\sqrt{3}\cdot\pi}{3}\cdot\frac{\sqrt{3}}{\sqrt{3}} = \frac{-4\pi R\cdot\cancel{3}}{\cancel{3}\sqrt{3}} \approx \underline{-7.26R < 0}$$ ✓

$\hookrightarrow$ Thus, there is a Maximum at $x = \sqrt{\frac{2}{3}}R$.

$$\bigstar - V\left(\sqrt{\frac{2}{3}}R\right) = \frac{\pi}{3}\left(\sqrt{\frac{2}{3}}R\right)^2 \cdot \sqrt{R^2 - \left(\sqrt{2/3}\,R\right)^2} = \frac{\pi}{3}\left(\frac{2}{3}\right)R^2 \cdot \sqrt{R^2 - \left(\frac{2}{3}\right)R^2}$$

$$= \left(\frac{2\pi R^2}{9}\right)\left(\sqrt{R^2(1 - \frac{2}{3})}\right) = \left(\frac{2\pi R^2}{9}\right)\left(R\sqrt{1/3}\right) = \boxed{\frac{2\pi R^3}{9\sqrt{3}}}$$ ✓

#59.) $c(x) = \frac{C(x)}{x} \longrightarrow$ Average Cost

$c'(x) = 0 \longrightarrow$ Minimum Cost

$C(x) = 16{,}000 + 200x + 4x^{3/2}$

a.) $c'(x) = 0 = \frac{d}{dx}\left(\frac{C(x)}{x}\right) = \left[\frac{(C'(x))(x) - (1)(C(x))}{x^2}\right]$

$$\Rightarrow \frac{xC'(x) - C(x)}{x^2} = 0 \longrightarrow xC'(x) - C(x) = 0$$

$$+C(x) \quad +C(x)$$

$$\frac{xC'(x)}{x} = \frac{C(x)}{x}$$

✓ $\boxed{C'(x) = \frac{C(x)}{x} \Rightarrow C'(x) = c(x)}$

↳ Marginal Cost is $C'(x)$.

⇒ Thus, Marginal Cost equals Average Cost $(c(x))$.

b.)(i) $x = 1000$

$$\hookrightarrow C(1000) = 16{,}000 + 200(1000) + 4(1000)^{3/2}$$

$$= 16{,}000 + 200{,}000 + 126491.1064$$

$$= \underline{342491.10}$$

$$\Rightarrow c(x) = \frac{C(x)}{x} \longrightarrow \frac{342491.10}{1000} = \underline{\underline{342}}$$

$$\Rightarrow c'(x) = 200 + 6x^{1/2}$$

$$\hookrightarrow 200 + 6\cdot\sqrt{1000} = \underline{\underline{390}}$$

☆ - Cost = \$342,491

Average Cost = \$342/unit

Marginal Cost = \$390/unit

(ii) $C'(x) = c(x)$

$$\overset{x\cdot}{\hookrightarrow} 200 + 6x^{1/2} = \frac{16{,}000 + 200x + 4x^{3/2}}{x} \cdot x$$

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→ **#59 Continued:**

$$6x\sqrt{x} + 200x = 16000 + 200x + 4x^{3/2} \quad ✓$$
$$\quad -200x \qquad\qquad -200x$$

$$6x\sqrt{x} = 16000 + 4x^{3/2}$$

$$\frac{6x\sqrt{x}}{4} = \frac{4(4000 + x^{3/2})}{4}$$

$$1.5x\sqrt{x} = 4000 + x^{3/2}$$

$$\hookrightarrow \left(1.5x^{1/2} \cdot x^{1/1} \Rightarrow 1.5x^{\frac{1}{2}+\frac{2}{2}} = \underline{\underline{1.5x^{3/2}}}\right)$$

$$1.5x^{3/2} = 4000 + x^{3/2}$$
$$-x^{3/2} \qquad\qquad -x^{3/2}$$

$$1.5x^{3/2} - x^{3/2} = 4000$$

$$x^{3/2}(1.5 - 1) = 4000$$

$$\frac{x^{3/2}(0.5)}{0.5} = \frac{4000}{0.5}$$

$$x^{3/2} = 8000 \longrightarrow \left(\sqrt[3]{x^3}\right)^2 = (8000)^2$$

$$x^3 = 64{,}000{,}000 \longrightarrow x = \sqrt{64{,}000{,}000}$$
$$= \underline{\underline{400}} \quad ✓$$

☆- In order to minimize the average cost, the $\boxed{\text{Production Level} = 400}$

(iii) $C(400) = 16{,}000 + 200(400) + 4(400)^{3/2}$

$= 16{,}000 + 80{,}000 + 32{,}000$

$= \underline{\underline{128{,}000}}$

$$c(400) = \frac{128{,}000}{400}$$

$= \underline{\underline{320}}$

☆- $\boxed{\text{Minimum Average Cost} = \$320/\text{unit}}$

OR

Average cost

$$A(x) = \frac{C(x)}{x}$$
$$= \frac{16000 + 200x + 4x^{3/2}}{x}$$
$$= 16000x^{-1} + 200 + 4x^{1/2}$$
$$\Rightarrow A'(x) = \frac{-16000}{x^2} + \frac{2}{\sqrt{x}} = 0$$
$$\Rightarrow \frac{2}{\sqrt{x}} = \frac{16(1000)}{x^2}$$
$$\Rightarrow 2x^2 = 4^2 10^3 \sqrt{x}$$
$$\Rightarrow \frac{4x^4}{4x} = \frac{4^4 10^6 x}{4x}$$
$$\Rightarrow x^3 = 4^3 10^6$$
$$\Rightarrow x = 4(10^2) =$$
$$= 400$$

#63.) * INITIAL SALES: 1200

* INITIAL PRICE: \$350

**Note: $\overset{x_1}{(1200}, \overset{y_1}{350)}$, $\overset{x_2}{(1280}, \overset{y_2}{340)}$

* SALES AFTER PRICE DECREASE: 1200 + 80 = 1280
* PRICE AFTER BEING LOWERED: \$350 − \$10 = \$340

a.)

$$m = \frac{y_2 - y_1}{x_2 - x_1} \Rightarrow \frac{340-350}{1280-1200} = -\frac{10}{80}$$

$$y - y_1 = m(x - x_1)$$

$$\hookrightarrow y - 350 = -\frac{10}{80}(x - 1200)$$

$$+350 \qquad +350$$

$$* \; p(x) = 350 - \frac{10}{80}(x-1200)$$

$$= 350 - \frac{10}{80}x + \frac{12000}{80} \longrightarrow 350 - \frac{1}{8}x + 150$$

$-\frac{1}{4}$ DEMAND FUNCTION $= \boxed{500 - \frac{1}{8}x}$ ← price , demand = x

b.) REVENUE: $R(x) = x\,p(x)$

$$\hookrightarrow x\left(500 - \frac{1}{8}x\right) = 500x - \frac{1}{8}x^2$$ ~~$= 0$~~

$$\Rightarrow R'(x) = 500 - \frac{1}{4}x = 0 \longrightarrow -4 \cdot -\frac{1}{4}x = -500 \cdot -4 \longrightarrow x = 2000$$

$$-500 \qquad -500$$

$$\Rightarrow R''(x) = -1/4 < 0 \; ✓$$

↳ Thus, there is a Maximum when x = 2000.

$$☆ - p(2000) = 500 - \frac{1}{8}(2000) = 500 - 250 = 250 \; ✓$$

↳ PRICE = $\boxed{\$250}$

c.) $C(x) = 35{,}000 + 120x \longrightarrow$ PROFIT: $P(x) = R(x) - C(x)$

$$\hookrightarrow P(x) = \left(500x - \frac{1}{8}x^2\right) - (35{,}000 + 120x)$$

$$= 500x - 1/8x^2 - 35{,}000 - 120x$$

$$= 380x - 1/8x^2 - 35{,}000$$

→

→ #63 <u>Continued</u>: $P(x) = -\frac{1}{8}x^2 + 380x - 35{,}000$

$$\Rightarrow P'(x) = -\frac{1}{4}x + \underset{-380}{380} = \underset{-380}{0} \longrightarrow {}^{-4}\cdot\ -\frac{1}{4}x = -380\cdot^{-4} \longrightarrow \underline{\underline{x = 1520}}$$

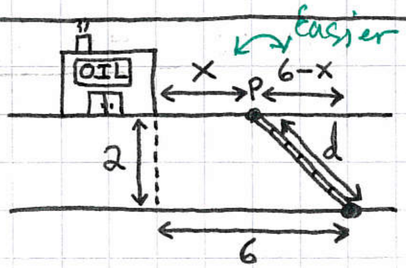
$$\Rightarrow P''(x) = \underline{\underline{-\frac{1}{4} < 0}}$$

↳ Thus, there is a Maximum at $x = 1520$

☆ - $p(1520) = 500 - \frac{1}{8}(1520) = 500 - 190 = \underline{\underline{310}}$

↳ PRICE = $\boxed{\$310}$ ✓

#51.)



* <u>Land Cost</u>: \$400,000/km ⇒ 400,000x

* <u>River Cost</u>: \$800,000/km ⇒ 800,000d

* <u>Interval</u>: $0 \leq x \leq 6$

$$\left.\begin{aligned} \Rightarrow d^2 &= (6-x)^2 + (2)^2 \\ &= 36 - 12x + x^2 + 4 \\ &= 40 - 12x + x^2 \end{aligned}\right\} \underline{\underline{d = \sqrt{x^2 - 12x + 40}}}$$ ✓

* Let (400,000) = 4 and (800,000) = 8 then

<u>COST</u>: $C(x) = 4x + 8\sqrt{x^2 - 12x + 40}$

$$\Rightarrow C'(x) = 4 + (8)(1/2)(x^2 - 12x + 40)^{-1/2}(2x - 12)$$ ✓

$$= 4 + \frac{8x - 48}{\sqrt{x^2 - 12x + 40}} \longrightarrow \frac{4\sqrt{x^2 - 12x + 40} + 8x - 48}{\sqrt{x^2 - 12x + 40}} = 0$$

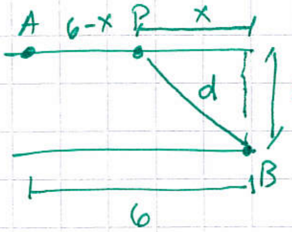
$$\hookrightarrow 4\sqrt{x^2 - 12x + 40} \underset{-(8x-48)}{+ 8x - 48} = \underset{-(8x-48)}{0} \longrightarrow \frac{4\sqrt{x^2 - 12x + 40}}{4} = \frac{-8x + 48}{4}$$

$$\hookrightarrow \sqrt{x^2 - 12x + 40} = -2x + 12 \longrightarrow x^2 - 12x + 40 = (-2x + 12)(-2x + 12)$$

$$\hookrightarrow \underset{-(x^2-12x+40)}{x^2 - 12x + 40} = \underset{-(x^2-12x+40)}{4x^2 - 48x + 144} \longrightarrow \underline{\underline{3x^2 - 36x + 104 = 0}}$$

⟶

Easier



$\Rightarrow d^2 = x^2 + 2^2$

$\Rightarrow C(x) = 4(6-x) + 8\sqrt{x^2+4}$

$C'(x) = -4 + \frac{8(x)}{\sqrt{x^2+4}} = 0$

$\Rightarrow 8x = 4\sqrt{x^2+4}$

$64x^2 = 16(x^2+4)$

$\rightarrow 48x^2 = 64$

$x^2 = \frac{4}{3}$

→ #51. Continued :

$$3x^2 - 36x + 104 = 0$$

$$\hookrightarrow \frac{36 \pm \sqrt{1296 - 4(3)(104)}}{2(3)} = \frac{36 \pm \sqrt{48}}{6} = \frac{36 \pm 4\sqrt{3}}{6}$$

$$\hookrightarrow 6 \pm \frac{2\sqrt{3}}{3} \approx \underline{\underline{4.85}} \, , \cancel{7.15}$$

↳ Beyond interval [0, 6].

FIRST DERIVATIVE TEST :

$$\Rightarrow C'(0) = \frac{4\sqrt{(0)^2 - 12(0) + 40} + 8((0) - 6)}{\sqrt{(0)^2 - 12(0) + 40}} = \frac{4\sqrt{40} - 48}{\sqrt{40}}$$

$$= \frac{8\sqrt{10} - 48}{2\sqrt{10}} = \frac{-22.7}{6.32} = \underline{\underline{-3.59}}$$

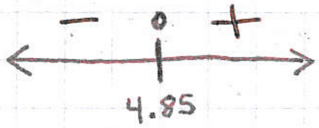
(−)

$$\Rightarrow C'(6) = \frac{4\sqrt{(6)^2 - 12(6) + 40} + 8((6) - 6)}{\sqrt{(6)^2 - 12(6) + 40}} = \frac{4\sqrt{36 - 72 + 40}}{\sqrt{36 - 72 + 40}}$$

$$= \frac{4\sqrt{4}}{\sqrt{4}} = \underline{\underline{4}}$$

(+)

⁂ $C'(x)$:



↳ Thus, there is a Minimum at $x \approx 4.85$

★ – In order to minimize the cost of the pipeline, point P must be $\boxed{\approx 4.85 \text{ km to the east of the refinery}}$