Continuity

- **Definition of Continuity:** A function f is *continuous at a number a* if $\lim_{x \to a} f(x) = f(a)$.
- **One-Sided Continuity:** A function f is *continuous from the right at a number a* if $\lim_{x \to a^+} f(x) = f(a)$ and f is *continuous from the left at a number a* if $\lim_{x \to a^-} f(x) = f(a)$.
- **Continuity in an Interval:** A function f is *continuous on an interval* if it is continuous at every number in the interval. (If f is defined only on one side of an endpoint of the interval, we understand *continuous* at the endpoint to mean *continuous from the right* or *continuous from the left.*)
- **Theorem 2.4:** If f and g are continuous at a and c is a constant, then the following functions are also continuous at a:

  1. $f + g$ **2.** $f - g$ **3.** cf
  4. fg **5.** f/g if $g(a) \neq 0$
- **Theorem 2.5:**
  (a) Any polynomial is continuous everywhere; that is, it is continuous on $\Re = (-\infty, \infty)$.
  (b) Any rational function is continuous wherever it is defined; that is, it continuous on its domain.
- **Theorem 2.7:** The following types of functions are continuous at every number in their domains:
  - polynomials
  - rational functions
  - root functions
  - trigonometric functions
  - inverse trigonometric functions
  - exponential functions
  - logarithmic functions
- **Theorem 2.8:** If f is continuous at b and $\lim_{x \to a} g(x) = b$, then $\lim_{x \to a} f(g(x)) = f(b)$.
  In other words, $\lim_{x \to a} f(g(x)) = f\left(\lim_{x \to a} g(x)\right)$
- **Theorem 2.9:** If g is continuous at a and f is continuous at $g(a)$, then the composite function $f \circ g$ given by $(f \circ g)(x) = f(g(x))$ is continuous at a.
- **The Intermediate Value Theorem:** Suppose that f is continuous on the closed interval $[a, b]$ and let N be any number between $f(a)$ and $f(b)$, where $f(a) \neq f(b)$. Then there exists a number c in (a, b) such that $f(c) = N$.