Sec 2.6: Limits at Infinity

- **Intuitive Definition of a Limit at Positive Infinity:** Let $f(x)$ be a function defined on some interval (a, ∞). Then

$$\lim_{x\to\infty} f(x) = L$$

means that the values of $f(x)$ can be made arbitrarily close to L by requiring x to be sufficiently large.

- **Intuitive Definition of a Limit at Negative Infinity:** Let $f(x)$ be a function defined on some interval $(-\infty, a)$. Then

$$\lim_{x\to-\infty} f(x) = L$$

means that the values of $f(x)$ can be made arbitrarily close to L by requiring x to be sufficiently large negative.

- **Horizontal Asymptotes:** The horizontal line $y = L$ is called a *horizontal asymptote* of the curve $y = f(x)$ if either

$$\lim_{x\to\infty} f(x) = L \qquad \text{or} \qquad \lim_{x\to-\infty} f(x) = L$$

- **Theorem 2.10:** If $r > 0$ is a rational number, then $\lim_{x\to\infty} \frac{1}{x^r} = 0$. If $r > 0$ is a rational number such that x^r is defined for all x, then $\lim_{x\to-\infty} \frac{1}{x^r} = 0$.

- **Precise Definition of a Limit at Positive Infinity:** Let f be a function defined on some open interval (a, ∞). Then

$$\lim_{x\to\infty} f(x) = L$$

if for every number $\epsilon > 0$ there is a corresponding N such that

$$\text{if } x > N \quad \text{then} \quad |f(x) - L| < \epsilon.$$

- **Precise Definition of a Limit at Negative Infinity:** Let f be a function defined on some open interval $(-\infty, a)$. Then

$$\lim_{x\to-\infty} f(x) = L$$

if for every number $\epsilon > 0$ there is a corresponding N such that

$$\text{if } x < N \quad \text{then} \quad |f(x) - L| < \epsilon.$$

- **Precise Definition of an Infinite Limit at Infinity:** Let f be a function defined on some open interval (a, ∞). Then

$$\lim_{x\to\infty} f(x) = \infty$$

means that for every positive number M there is a corresponding positive number N such that

$$\text{if } x > N \quad \text{then} \quad f(x) > M.$$