Sec 4.7: Optimization Problems

- **Steps in Solving Optimization Problems:**

  1. **Understand the Problem.** The first step in to read the problem carefully until it is clearly understood. Ask yourself: *What is unknown? What are the given quantities? What are the conditions?*
  2. **Draw a Diagram.** In most problems it is useful to draw a diagram and identify the given and required quantities on the diagram.
  3. **Introduce Notation.** Assign a symbol to the quantity that is to be maximized or minimized (let's call it Q for now). Also select symbols $(a, b, c, \ldots, x, y)$ for other unknown quantities and label the diagram with these symbols. It may help to use initials as suggestive symbols (like, h for height or t for time).
  4. Express Q in terms of some of the other symbols from Step 3.
  5. If Q has been expressed as a function of more than one variable in Step 4, use the given information to find relationships (in the form of equations) among these variables. Then use these equations to eliminate all but one of the variables in the expression for Q, say x. Thus $Q = f(x)$.
  6. Use the methods of Sections 4.1 and 4.3 to find the absolute maximum or minimum value of $f(x)$.

- **Example 1:** A farmer has 2400 ft of fencing and wants to fence off a rectangular field that borders a straight river. He needs no fence along the river. What are the dimensions of the field that has the largest area?

- **Example 2:** A cylindrical can is to be made to hold 1 liter of oil. Find the dimensions that will minimize the cost of the metal to manufacture the can.

- **Example 3:** Find the point on the parabola $y^2 = 2x$ that is closest to the point (1,4).

- **Example 4:** A man launches his boat from a point A on a bank of a straight river, 3 km wide, and wants to reach point B, 8 km downstream on the opposite bank, as quickly as possible. If he can row 6 km/hr and run 8 km/hr, where should he land to reach B as soon as possible?

- **Example 5:** Find the area of the largest rectangle that can be inscribed in a semicircle of radius r.

- **Example 6:** A store has been selling 200 flat-screen TVs a week at \$350 each. A market survey indicates that for each \$10 rebate offered to buyers, the number of TVs sold will increase by 20 a week. Find the demand function and the revenue function. How large a rebate should the store offer to maximize its revenue?