Chapter 4: Applications of Differentiation

- **Absolute/Global Extrema:** Let c be a number in the domain D of a function f. Then $f(c)$ is the
  - **absolute maximum** value of f on D if $f(c) \geq f(x)$ for all x in D.
  - **absolute minimum** value of f on D if $f(c) \leq f(x)$ for all x in D.
- **Local Extrema:** The number $f(c)$ is the
  - **local maximum** value of f on D if $f(c) \geq f(x)$ when x is near c.
  - **local minimum** value of f on D if $f(c) \leq f(x)$ when x is near c.
- **The Extreme Value Theorem:** If f is continuous on a closed interval $[a, b]$, then f attains an absolute maximum value $f(c)$ and an absolute minimum value $f(d)$ at some numbers c and d in $[a, b]$.
- **Fermat's Theorem:** If f has a local maximum or minimum at c, and if $f'(c)$ exists, then $f'(c) = 0$.
- **Critical Number:** A *critical number* of a function f is a number c in the domain of f such that either $f'(c) = 0$ or $f'(c)$ does not exist.
- **Fermat's Theorem rephrased:** If f has a local maximum or minimum at c, then c is a critical number of f.
- **The Closed Interval Method:** To find the *absolute* maximum and minimum values of a continuous function f on a closed interval $[a, b]$
  1. Find the values of f at the critical numbers of f in (a, b).
  2. Find the values of f at the endpoints of the interval.
  3. The largest of the values from Steps 1 and 2 is the absolute maximum value; the smallest of the values from Steps 1 and 2 is the absolute minimum value.
- **Rolle's Theorem:** Let f be a function that satisfies the following three hypotheses:
  1. f is continuous on the closed $[a, b]$.
  2. f is differentiable on the open (a, b).
  3. $f(a) = f(b)$

  Then there is a number c in (a, b) such that $f'(c) = 0$.
- **The Mean Value Theorem:** Let f be a function that satisfies the following three hypotheses:
  1. f is continuous on the closed $[a, b]$.
  2. f is differentiable on the open (a, b).

  Then there is a number c in (a, b) such that $f'(c) = \dfrac{f(b) - f(a)}{b - a}$ or, equivalently, $f(b) - f(a) = f'(c)(b - a)$.
- **Theorem 4.2.5:** If $f'(x) = 0$ for all x in an interval (a, b), then f is constant on (a, b).
- **Corollary 4.2.7:** If $f'(x) = g'(x)$ for all x in an interval (a, b), then $f - g$ is constant on (a, b); that is, $f(x) = g(x) + c$ were c is a constant.

- **Increasing/Decreasing Test:**

  1. If $f'(x) > 0$ on an interval, then f is increasing on that interval.

  2. If $f'(x) < 0$ on an interval, then f is decreasing on that interval.

- **The First Derivative Test:** Suppose that c is a critical number of a continuous function f.

  1. If $f'(x)$ changes from positive to negative at c, then f has a local maximum at c.

  2. If $f'(x)$ changes from negative to positive at c, then f has a local minimum at c.

  3. If $f'(x)$ does not change sign at c, then f has no a local maximum or minimum at c.

- **Concavity:** If a graph of f lies above all of its tangents on an interval I, then it is called *concave upward* on I. If a graph of f lies below all of its tangents on an interval I, then it is called *concave downward* on I.

- **Concavity Test:**

  1. If $f''(x) > 0$ for all x in I, then the graph of f is concave upward on I.

  2. If $f''(x) < 0$ for all x in I, then the graph of f is concave downward on I.

- **Inflection Point:** A point P on a curve $y = f(x)$ is called an *inflection point* if f is continuous there and the curve changes from concave upward to concave downward or from concave downward to concave upward at P.

- **The Second Derivative Test:** Suppose f'' is continuous near c.

  1. If $f'(c) = 0$ and $f''(c) > 0$, then f has a local minimum at c.

  2. If $f'(c) = 0$ and $f''(c) < 0$, then f has a local maximum at c.

Section 4.4: Indeterminate Forms and L'Hospital's Rule

- **L'Hospital's Rule:** Suppose f and g are differentiable at $g'(x) \neq 0$ on an open interval I that contains a (except possibly at a). Suppose that

$$\lim_{x\to a} f(x) = 0 \qquad \text{and} \qquad \lim_{x\to a} g(x) = 0$$

or that

$$\lim_{x\to a} f(x) = \pm\infty \qquad \text{and} \qquad \lim_{x\to a} g(x) = \pm\infty$$

(In other words, we have an indeterminate form of type $\frac{0}{0}$ or $\frac{\infty}{\infty}$.) Then

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$$

if the limit on the right side exists (or is ∞ or $-\infty$).