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Instructions: *Include all relevant work to get full credit.*

Quiz #10

1. Find the linear approximation of the function $f(x) = \sqrt[3]{1+x}$ at $a = 0$ and use it to approximate the value of $\sqrt[3]{0.95}$ [3]

$\Rightarrow \quad y = L(x) = f(a) + f'(a)(x-a)$ $\qquad f'(x) = \frac{1}{3}(1+x)^{-2/3}$

$= f(0) + f'(0)(x)$ $\qquad \Rightarrow f'(0) = \frac{1}{3}$

$= 1 + \frac{1}{3}(x)$

Note that

$\sqrt[3]{0.95} = f(-.05) \approx 1 + \frac{1}{3}(-.05) = 1 - \frac{5}{300} = \frac{295}{300} = \frac{59}{60}$

2. Find the differential of $y = \theta^2 \sin 2\theta$. [2]

$\Rightarrow \quad dy = \left(2\theta^2 \cos 2\theta + 2\theta \sin 2\theta\right) d\theta$

3. Find the differential dy if $y = e^{5x^3}$ and evaluate its value when $x = 0$ and $dx = 0.1$ [2]

$\Rightarrow \quad dy = e^{5x^3}(15x^2)\,dx$

$= e^0(5)(.1) = .5$

4. The radius of a circular disk is given as 24 cm with a maximum error in measurement of 0.2 cm. Use differentials to estimate the maximum error in the calculated area of the disk. What is the relative error? [3]

$A = \pi r^2 \longrightarrow dA = 2\pi r\, dr$

$\Rightarrow$ max error for $A = 2\pi(24)(.2) = 9.6\pi$ cm^2

$\Rightarrow$ Relative Error $= \frac{dA}{A} = \frac{2\pi r\, dr}{\pi r^2} = 2\frac{dr}{r}$

$\Rightarrow$ Percentage Error $= \frac{9.6\pi}{(24^2)\pi} \approx \frac{1}{60} \overset{11}{\longleftrightarrow} 2\left(\frac{.2}{24}\right) = \frac{.4}{24} = \frac{1}{60}$

$\approx 1.67\%$