

Quiz #11

1. Determine the exact value of the following: (Show your complete work to get full credit.)

a. $\tanh(\ln 2)$

$$\tanh(x) = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}} \quad \text{Note: } -\ln 2 = \ln 2^{-1} = \ln \frac{1}{2}$$

$$\Rightarrow \tanh(\ln 2) = \frac{e^{\ln 2} - e^{-\ln 2}}{e^{\ln 2} + e^{-\ln 2}} = \frac{2 - \frac{1}{2}}{2 + \frac{1}{2}} = \frac{\frac{3}{2}}{\frac{5}{2}} = \frac{3}{5}$$

b. $\tanh^{-1}(\frac{1}{2})$

$$\Rightarrow \tanh^{-1}(x) = \frac{1}{2} \ln\left(\frac{1+x}{1-x}\right)$$

$$\Rightarrow \tanh^{-1}\left(\frac{1}{2}\right) = \frac{1}{2} \ln\left(\frac{1+\frac{1}{2}}{1-\frac{1}{2}}\right) = \frac{1}{2} \ln\left(\frac{\frac{3}{2}}{\frac{1}{2}}\right) = \frac{1}{2} \ln 3$$

2. Find the derivative of the following:

a. $f(x) = \sinh^2(5x) + \operatorname{sech}(5x)$

$$\Rightarrow f'(x) = 2(\sinh(5x) \cosh(5x)(5)) - \operatorname{sech}(5x) \tanh(5x) 5$$

b. $g(x) = \ln(\cosh 3x) + e^{\coth 2x}$

$$\Rightarrow g'(x) = \frac{3 \sinh 3x}{\cosh 3x} + e^{\coth(2x)} (-\operatorname{csch}^2(2x))(2)$$

c. $y = \cosh^{-1}(\sinh x) + \tanh^{-1} \sqrt{x}$

$$y' = \frac{1}{\sqrt{\sinh^2 x - 1}} \cdot \cosh x + \frac{1}{1 - (\sqrt{x})^2} \cdot \frac{1}{2\sqrt{x}}$$