

Instructions: Include all relevant work to get full credit. Write your solutions using proper notations. Encircle your final answers.

Quiz #12

1. Find the critical numbers of the function:

a. $f(x) = x^3 + 6x^2 - 15x$

[3]

$$\rightarrow f'(x) = 3x^2 + 12x - 15 = 0$$

$$3(x^2 + 4x - 5) = 0$$

$$3(x+5)(x-1) = 0$$

$$x = -5 \mid x = 1$$

b. $g(x) = x^{4/5}(x-4)^2$

[3]

$$\rightarrow g'(x) = x^{4/5} 2(x-4) + \frac{4}{5} x^{-1/5} (x-4)^2$$

$$= \frac{2}{5} (x)^{-1/5} (x-4) [5x + 2(x-4)]$$

$$= \frac{2}{5} \frac{(7x-8)(x-4)}{\sqrt[5]{x}} \rightarrow \text{critical values: } \{4, 8/7, 0\}$$

2. Find the absolute maximum and absolute minimum values of $f(x) = xe^{2x}$ on $[-2, 1]$.

[4]

$$\rightarrow f'(x) = x e^{2x} (2) + e^{2x} = e^{2x} (2x+1) = 0$$

$$\Rightarrow x = -1/2$$

$$\rightarrow f(-1/2) = -\frac{1}{2} e^{-1} \approx -0.184 \quad \leftarrow \text{abs. min}$$

$$f(-2) = -2 e^{-4} \approx -0.0366$$

$$f(1) = e^2 \quad \leftarrow \text{abs. max.}$$

Bonus: State the Extreme Value Theorem. You may use your own words.

[1]

If $f(x)$ is continuous on $[a, b]$, then $f(x)$ ~~has~~ attains an abs. max and an abs. min somewhere in $[a, b]$.