

Instructions: Include all relevant work to get full credit. Write your solutions using proper notations. Encircle your final answers.

Quiz #13

1. Find c that satisfies the conclusion of the Mean Value Theorem for $f(x) = x^3 - 3x + 2$ on $[-2, 2]$. [3]

$$\Rightarrow f'(x) = 3x^2 - 3 = \frac{f(b) - f(a)}{b - a} = \frac{f(2) - f(-2)}{2 - (-2)}$$

$$= \frac{4 - 0}{4} = 1$$

$$\Rightarrow f'(c) = 3c^2 - 3 = 1$$

$$\Rightarrow c^2 = \frac{4}{3} \Rightarrow c = \pm \sqrt{\frac{4}{3}}$$

2. Consider the function $f(x) = x^4 - 4x^3$. Construct the first derivative chart that shows the critical values and where $f'(x)$ is positive and negative. Also, include in this chart where the $f(x)$ is increasing and decreasing. Then specify if there is any local maximum and/or local minimum points. [4]

$$f'(x) = 4x^3 - 12x^2 = 0$$

$$\Rightarrow 4x^2(x - 3) = 0$$

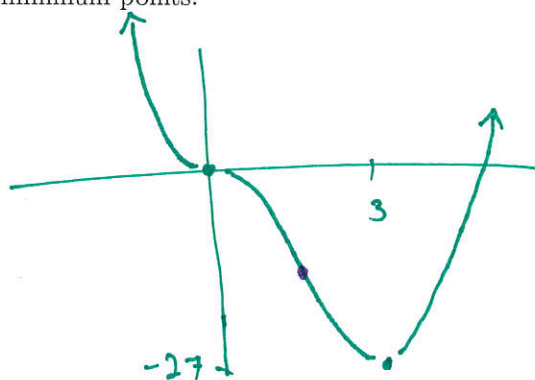
$$x = 0 \quad | \quad x = 3$$

$4x^2$	+	0	+	+
$(x-3)$	-	-	0	+

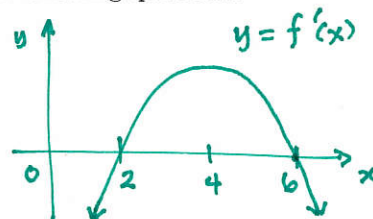
$f'(x)$	-	0	-	3	+
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$f(x)$	dec		dec		inc
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→ local min



3. Use the graph of the derivative (shown below) to answer the following questions:



- a. What are the critical values? [1]

$$x = 2, 6$$

- b. For what values of x is $f(x)$ decreasing? Give your answer in interval notation. [1]

$$= (-\infty, 2) \cup (6, \infty)$$

- c. At what value of x does $f(x)$ have a local maximum? [1]

$$x = 6$$

Bonus: State the Mean Value Theorem. [1]

If $f(x)$ is cont. on $[a, b]$ and differentiable on (a, b) ,
then $\exists c$ in (a, b) s.t. $f'(c) = \frac{f(b) - f(a)}{b - a}$.