

Instructions: Include all relevant work to get full credit. Write your solutions using proper notations. Encircle your final answers.

Quiz #14

1. Evaluate the following limits using the L'Hospital Rule:

a. $\lim_{x \rightarrow 0} \frac{x^2}{\cos(2x) - 1}$ $\frac{\rightarrow 0}{\rightarrow 0}$ [2]

$$\stackrel{L'H}{=} \lim_{x \rightarrow 0} \frac{2x}{-2\sin(2x)} \quad \frac{\rightarrow 0}{\rightarrow 0}$$

$$\stackrel{L'H}{=} \lim_{x \rightarrow 0} \frac{2}{-4\cos(2x)} = \frac{2}{-4} = -\frac{1}{2}$$

b. $\lim_{x \rightarrow 0} \frac{e^x - (1+x)^2}{x^2 + 3x}$ $\frac{\rightarrow 1-1}{\rightarrow 0+0} = \frac{\rightarrow 0}{\rightarrow 0}$ [2]

$$\stackrel{L'H}{=} \lim_{x \rightarrow 0} \frac{e^x - 2(1+x)}{2x + 3} = \frac{e^0 - 2(1+0)}{0 + 3}$$

$$= \frac{1-2}{3} = -\frac{1}{3}$$

c. $\lim_{x \rightarrow \infty} x \tan\left(\frac{2}{x}\right)$ $(+\infty)(\rightarrow 0)$ [3]

$$= \lim_{x \rightarrow \infty} \frac{\tan\left(\frac{2}{x}\right)}{\frac{1}{x}} \stackrel{L'H}{=} \lim_{x \rightarrow \infty} \frac{2\sec^2\left(\frac{2}{x}\right) \cdot \left(-\frac{1}{x^2}\right)}{-\frac{1}{x^2}}$$

$$= \lim_{x \rightarrow \infty} 2\sec^2\left(\frac{2}{x}\right) = 2\sec^2(0)$$

$$= 2.$$

d. $\lim_{x \rightarrow 0^+} x^{\sqrt{x}}$ $= \lim_{x \rightarrow 0^+} e^{\ln x^{\sqrt{x}}} = e^0 = 1.$ [3]

Note: $\lim_{x \rightarrow 0^+} \ln x^{\sqrt{x}} = \lim_{x \rightarrow 0^+} \sqrt{x} \ln x = \lim_{x \rightarrow 0^+} \frac{\ln x}{\frac{1}{\sqrt{x}}}$

$$\stackrel{L'H}{=} \lim_{x \rightarrow 0^+} \frac{\frac{1}{x}}{-\frac{1}{2}x^{-3/2}} = \lim_{x \rightarrow 0^+} -2x^{1/2} = 0$$