

Instructions: Include all relevant work to get full credit. Write your solutions using proper notations. Encircle your final answers.

Quiz #15

1. Find the general antiderivative of the following functions:

a. $f(x) = 8x^3 - x + 3$

[2]

$$\begin{aligned}\Rightarrow F(x) &= \int (8x^3 - x + 3) dx = \frac{8x^4}{4} - \frac{x^2}{2} + 3x + C \\ &= 2x^4 - \frac{1}{2}x^2 + 3x + C\end{aligned}$$

b. $f(\theta) = \sec^2(\theta) - \sec(\theta)\tan(\theta)$

[2]

$$\begin{aligned}\Rightarrow F(\theta) &= \int (\sec^2\theta - \sec\theta\tan\theta) d\theta \\ &= \int \sec^2\theta d\theta - \int \sec\theta\tan\theta d\theta \\ &= \tan\theta - \sec\theta + C\end{aligned}$$

c. $f(t) = t^2(7\sqrt{t} - \frac{2}{t^3})$

[2]

$$\begin{aligned}\Rightarrow F(t) &= \int t^2(7\sqrt{t} - \frac{2}{t^3}) dt = \int t^2(7t^{1/2} - 2t^{-3}) dt \\ &= 7 \int t^{5/2} dt - 2 \int \frac{1}{t} dt \\ &= 7 \cdot \frac{t^{7/2}}{7/2} - 2 \ln|t| + C \\ &= 2t^{7/2} - 2 \ln|t| + C\end{aligned}$$

2. Find $f(x)$ if $f'(x) = \frac{x+1}{\sqrt{x}}$ and $f(1) = 5$.

[4]

$$\begin{aligned}\Rightarrow f(x) &= \int \frac{x+1}{\sqrt{x}} dx = \int (x^{1/2} + x^{-1/2}) dx \\ &= \frac{x^{3/2}}{3/2} + \frac{x^{1/2}}{1/2} + C\end{aligned}$$

$$\begin{aligned}\Rightarrow f(1) &= \frac{2}{3}(1)^{3/2} + 2(1)^{1/2} + C = 5 \Rightarrow \frac{2}{3} + 2 + C = 5 \\ &\Rightarrow C = 3 - \frac{2}{3} \\ &= \frac{7}{3}\end{aligned}$$

$$\therefore f(x) = \frac{2}{3}x^{3/2} + 2\sqrt{x} + \frac{7}{3}$$