

Instructions: Include all relevant work to get full credit. Write your solutions using proper notations. Encircle your final answers.

Quiz #16

1. Use the substitution method to find the general antiderivative of the following: (Make sure that your final answer is in terms of the original variable.)

$$\begin{aligned} \text{a. } \int x e^{x^2+1} dx &= \int e^u \left(\frac{1}{2} du\right) = \frac{1}{2} \int e^u du & [2] \\ \text{Let } u &= x^2+1 & = \frac{1}{2} e^u + C \\ du &= 2x dx & = \frac{1}{2} e^{x^2+1} + C \end{aligned}$$

$$\begin{aligned} \text{b. } \int 4x \sqrt[3]{x^2-3} dx &= \int 2 \sqrt[3]{u} du = 2 \int u^{1/3} du & [2] \\ \text{Let } u &= x^2-3 & = 2 \frac{(u^{4/3})}{4/3} + C \\ du &= 2x dx & = \frac{3}{2} (x^2-3)^{4/3} + C \end{aligned}$$

$$\begin{aligned} \text{c. } \int \frac{3t^2-1}{t^3-t} dt &= \int \frac{1}{u} du = \ln|u| + C & [2] \\ \text{Let } u &= t^3-t & = \ln|t^3-t| + C \\ du &= (3t^2-1) dt \end{aligned}$$

$$\begin{aligned} \text{d. } \int \frac{\sec^2 \theta}{\tan^2 \theta} d\theta &= \int \frac{du}{u^2} = \int u^{-2} du = -u^{-1} + C & [2] \\ \text{Let } u &= \tan \theta & = -\frac{1}{\tan \theta} + C \\ du &= \sec^2 \theta d\theta \end{aligned}$$

$$\begin{aligned} \text{e. } \int 4x^3 \sqrt[3]{x^2-3} dx &= \int 2x^2 \sqrt[3]{x^2-3} (2x dx) & [2] \\ \text{Let } u &= x^2-3 & = \int 2(u+3) u^{1/3} du \\ du &= 2x dx & = \int 2u^{4/3} + 6u^{1/3} du \\ \hookrightarrow x^2 &= u+3 & = \frac{2u^{7/3}}{7/3} + \frac{6u^{4/3}}{4/3} + C \\ & & = \frac{6}{7} (x^2-3)^{7/3} + \frac{9}{2} (x^2-3)^{4/3} + C \end{aligned}$$