NAME _Colin Griffin_   AUGUST 8, 2018
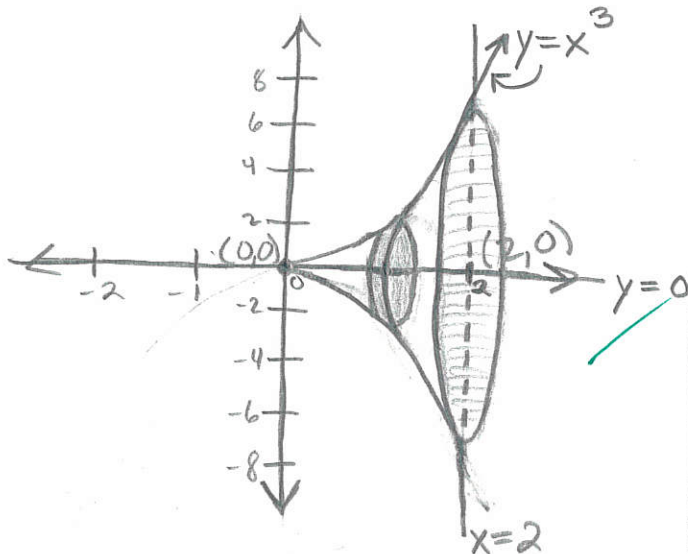
Instructions: *Include all relevant work to get full credit. Write your solutions using proper notations. Encircle your final answers.*

Quiz #20

1. Find the volume of the solid obtained by rotating the region bounded by $y = x^3$, $y = 0$, and $x = 2$ about the $x - axis$.   [5]

$y = x^3$ | $x = 2$ | Rotate about
$y = 0$ | | x-axis

$* \ V = \int_a^b A(x)\,dx$

$* \ A(x) = \pi r^2$

$\qquad \hookrightarrow \pi(x^3)^2$

$\Rightarrow V = \int_0^2 (\pi x^6)\,dx = \pi \int_0^2 x^6\,dx$

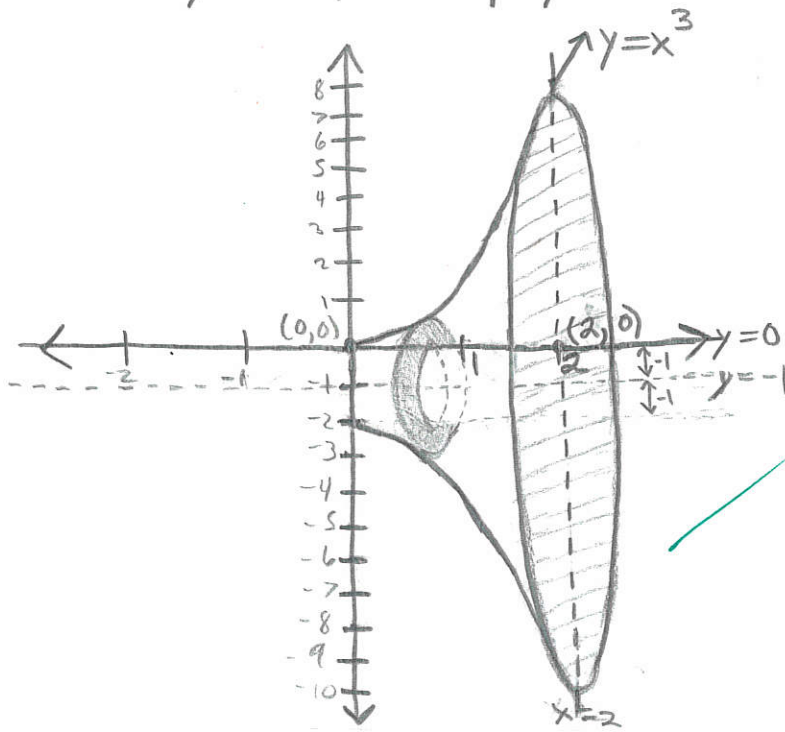$\qquad = \pi \left(\frac{x^7}{7}\right)\Big|_0^2$

$\Rightarrow \pi\left(\frac{(2)^7}{7}\right) - \pi\left(\frac{(0)^7}{7}\right) = \pi\left(\frac{128}{7}\right) - 0$

$\qquad\qquad = \boxed{\frac{128}{7}\pi}$

2. Find the volume of the solid obtained by rotating the region bounded by $y = x^3$, $y = 0$, and $x = 2$ about the line $y = -1$.   [5]

$y = x^3$ | $x = 2$ | Rotate about
$y = 0$ | | $y = -1$

$* \ V = \int_a^b A(x)\,dx$

$* \ A(x) = \pi r^2 \Rightarrow \pi\left[\left(\begin{smallmatrix}OUTER\\RADIUS\end{smallmatrix}\right)^2 - \left(\begin{smallmatrix}INNER\\RADIUS\end{smallmatrix}\right)^2\right]$

$\Rightarrow O.R = x^3 - (-1) = x^3 + 1$

$\Rightarrow I.R. = 0 - (-1) = 1$

$\Rightarrow V = \pi\int_0^2 \left[(x^3+1)^2 - (1)^2\right]\,dx = \pi\int_0^2 \left[x^6 + 2x^3 + 1 - 1\right]\,dx$

$\qquad = \pi\int_0^2 (x^6 + 2x^3)\,dx = \pi\left(\frac{x^7}{7} + \frac{x^4}{2}\right)\Big|_0^2$

$\Rightarrow \pi\left(\frac{(2)^7}{7} + \frac{(2)^4}{2}\right) - \pi\left(\frac{(0)^7}{7} + \frac{(0)^4}{2}\right)$

$\qquad = \pi\left(\frac{128}{7} + \frac{16}{2}\right) = \pi\left(\frac{256 + 112}{14}\right) = \boxed{\frac{184}{7}\pi}$