

**Instructions:** Include all relevant work to get full credit. Write your solutions using proper notations. Encircle your final answers.

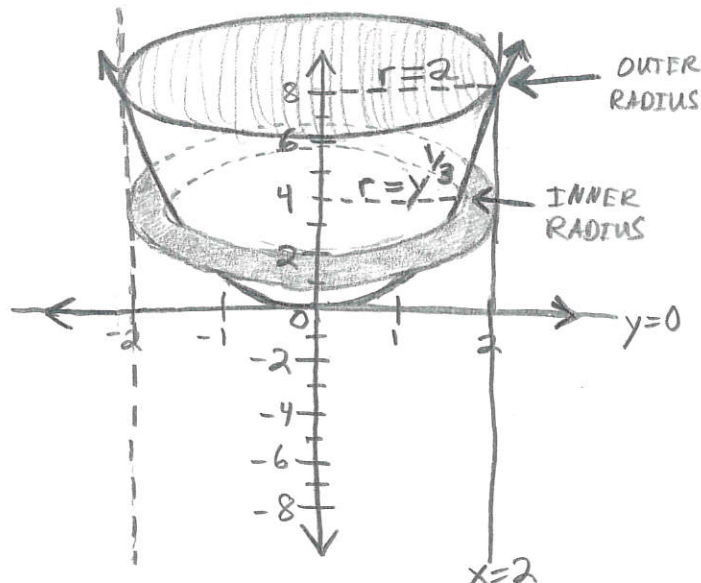
## Quiz #21

$\frac{10}{10}$  Niu work!

1. Find the volume of the solid obtained by rotating the region bounded by  $y = x^3$ ,  $y = 0$ , and  $x = 2$  about the  $y$ -axis. [5]

$$y = x^3 \rightarrow x = y^{1/3} \quad \left| \begin{array}{l} \text{Rotate about} \\ y\text{-axis} \end{array} \right.$$

$$y = 0 \quad \left| \quad x = 2 \right.$$



$$* V = \int_a^b A(y) dy$$

$$* A(y) = \pi r^2 \Rightarrow \pi \left[ (\text{OUTER RADIUS})^2 - (\text{INNER RADIUS})^2 \right]$$

$$\Rightarrow V = \int_0^8 \pi \left[ (2)^2 - (y^{1/3})^2 \right] dy = \pi \int_0^8 (4 - y^{2/3}) dy$$

$$= \pi \left( 4y - \frac{3y^{5/3}}{5} \right) \Big|_0^8$$

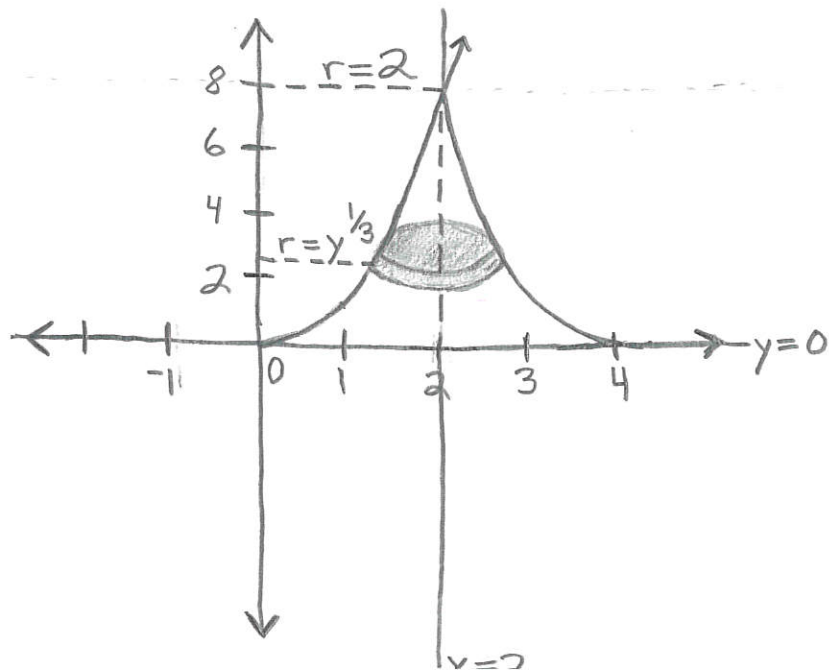
$$\Rightarrow \pi \left( 4(8) - \frac{3(8)^{5/3}}{5} \right) - \pi \left( 4(0) - \frac{3(0)^{5/3}}{5} \right)$$

$$= \pi \left( 32 - \frac{96}{5} \right) = \pi \left( \frac{160 - 96}{5} \right) = \boxed{\frac{64}{5} \pi}$$

2. Find the volume of the solid obtained by rotating the region bounded by  $y = x^3$ ,  $y = 0$ , and  $x = 2$  about the line  $x = 2$ . [5]

$$y = x^3 \rightarrow x = y^{1/3} \quad \left| \begin{array}{l} \text{Rotate about} \\ x = 2 \end{array} \right.$$

$$y = 0 \quad \left| \quad x = 2 \right.$$



$$* V = \int_a^b A(y) dy$$

$$* A(y) = \pi r^2$$

$$\hookrightarrow \pi (2 - y^{1/3})^2$$

$$\Rightarrow V = \int_0^8 \pi \left[ 4 - 4y^{1/3} + y^{2/3} \right] dy$$

$$= \pi \left( 4y - 3y^{4/3} + \frac{3y^{5/3}}{5} \right) \Big|_0^8$$

$$\Rightarrow \pi \left( 4(8) - 3(8)^{4/3} + \frac{3(8)^{5/3}}{5} \right) - \pi \left( 4(0) - 3(0)^{4/3} + \frac{3(0)^{5/3}}{5} \right)$$

$$= \pi \left( 32 - 48 + \frac{96}{5} \right) = \pi \left( \frac{96 - 80}{5} \right) = \boxed{\frac{16}{5} \pi}$$