

Quiz #2

1. Find the inverse of $f(x) = \frac{2e^x}{3+e^x}$. [2]

$$\begin{aligned} \Rightarrow y &= \frac{2e^x}{3+e^x} \Rightarrow 3y + ye^x = 2e^x \\ &\Rightarrow 3y = 2e^x - ye^x \\ &\Rightarrow 3y = e^x(2-y) \\ &\Rightarrow \frac{3y}{2-y} = e^x \Rightarrow x = \ln\left(\frac{3y}{2-y}\right) \Rightarrow f^{-1}(x) = \ln\left(\frac{3x}{2-x}\right) \end{aligned}$$

2. Solve each equation for x .

a. $3^{7-3x} = 9 = 3^2$ [2]

$$\Rightarrow 7-3x = 2$$

$$\Rightarrow -3x = -5$$

$$\Rightarrow \boxed{x = \frac{5}{3}}$$

$$\log_3 3^{7-3x} = \log_3(3^2) = 2$$

$$7-3x = 2$$

b. $\ln(2x) - \ln(1-x) = \ln 2$ [2]

$$\Rightarrow \ln\left(\frac{2x}{1-x}\right) = \ln 2$$

$$\Rightarrow \frac{2x}{1-x} = 2$$

$$\Rightarrow 2x = 2 - 2x$$

$$\Rightarrow 4x = 2$$

$$\Rightarrow \boxed{x = \frac{1}{2}}$$

3. Express $f(x) = 3\ln(x+2) + \ln(x+3) - 2\ln(x-2)$ as a single logarithm. [2]

$$\begin{aligned} \Rightarrow f(x) &= \ln(x+2)^3 + \ln(x+3) - \ln(x-2)^2 \\ &= \ln\left[\frac{(x+2)^3(x+3)}{(x-2)^2}\right] \end{aligned}$$

4. Determine the value of $\cot^2\left(\frac{25\pi}{3}\right)$. [2]

$$\cot^2\left(\frac{25\pi}{3}\right) = \cot^2\left(\frac{\pi}{3}\right) = \frac{\cos^2(\pi/3)}{\sin^2(\pi/3)} = \frac{(\frac{1}{2})^2}{(\frac{\sqrt{3}}{2})^2} = \frac{1}{3}$$

$$= \cot^2\left(\frac{25\pi}{3} - \frac{24\pi}{3}\right)$$