NAME ______________________ JUNE 27, 2018

Quiz #4

1. Use the Squeeze Theorem to find $\lim_{x\to 0} |x|\sin^2\left(\frac{1}{x}\right)$. [2]

Note: $0 \le \sin^2\left(\frac{1}{x}\right) \le 1 \implies 0 \le |x|\sin^2\left(\frac{1}{x}\right) \le |x|$

Since $\lim_{x\to 0} 0 = 0$ and $\lim_{x\to 0} |x| = 0$, then by the Squeeze Theorem, $\lim_{x\to 0} |x|\sin^2\left(\frac{1}{x}\right) = 0$.

2. Use the Intermediate Value Theorem to argue that $f(x) = x^3 - 2x$ has a root in the closed interval $[1, 2]$. Recall that $f(x)$ has a root at $x = c$ if $f(c) = 0$. [2]

Note that $f(x) = x^3 - 2x$ is a polynomial, therefore it is continuous on $[1, 2]$. Also, $f(1) = -1$ and $f(2) = 4$. So by the IVT, $\exists\, c$ in $(1,2)$ s.t. $f(c) = 0$.

3. Evaluate the limit, if it exists. If the limit does not exist, write ∞, $-\infty$, or **DNE**.

a. $\lim_{x\to -\infty} \frac{3x^2 - 3x + 2}{4x - 2x^2}$ $= \lim_{x\to -\infty} \frac{3 - \frac{3}{x} + \frac{2}{x^2}}{\frac{4}{x} - 2} = -\frac{3}{2}$ [2]

b. $\lim_{t\to\infty} \frac{\sqrt{t^2+4} - 2}{5t}$ $= \lim_{t\to\infty} \frac{\sqrt{1 + \frac{4}{t^2}} - \frac{2}{t}}{5} = \frac{1}{5}$ [2]

c. $\lim_{t\to\infty}\left(\sqrt{t^2+4t} - t\right) \cdot \frac{\left(\sqrt{t^2+4t} + t\right)}{\sqrt{t^2+4t} + t}$ $= \lim_{t\to\infty} \frac{\cancel{t^2} + 4t - \cancel{t^2}}{\sqrt{t^2+4t} + t}$ [2]

$= \lim_{t\to\infty} \frac{4}{\sqrt{1+\frac{4}{t}} + 1} = \frac{4}{2} = 2$