NAME ____________________________ JUNE 29, 2018

Instructions: *Include all relevant work to get full credit.*

Quiz #5

1. Calculate the slope of the tangent line to the curve $y = 3\sqrt{x}$ at $x = 4$ using the limit definition. Then obtain the equation of the tangent lint. [3]

$$m = \lim_{x\to 4} \frac{f(x) - f(a)}{x-a} = \lim_{x\to 4} \frac{3\sqrt{x} - 3\sqrt{4}}{x-4} = \lim_{x\to 4} \frac{3(\cancel{\sqrt{x}-2})}{(\cancel{\sqrt{x}-2})(\sqrt{x}+2)} = \frac{3}{4}$$

when $x = 4$, $y = 3\sqrt{4} = 6 \Rightarrow 6 = \frac{3}{4}(4) + b \to b = 3$

$\Rightarrow$ Equation of the T.L is $y = \frac{3}{4}x + 3$

2. If a ball is thrown into the air with an initial velocity of 60 ft/s, its height (in feet) after t seconds is given by $y = 60t - 3t^2$.

a. Use the limit definition to derive the (instantaneous) velocity function $v(a)$. [2]

$$v(a) = \lim_{t\to a} \frac{f(t) - f(a)}{t-a} = \lim_{t\to a} \frac{(60t - 3t^2) - (60a - 3a^2)}{t-a}$$

$$= \lim_{t\to a} \frac{60(t-a) - 3(t^2 - a^2)}{(t-a)}$$

$$= \lim_{t\to a} \frac{\cancel{(t-a)}(60 - 3(t+a))}{\cancel{(t-a)}} = 60 - 6a$$

b. Use the function you obtained in part (a) to evaluate the velocity at time $a = 12$ seconds. Explain the meaning of this value in the context of the problem. Is the rock moving upwards to downwards at that time? [2]

$V(12) = 60 - 6(12) = -12$ ft/s

This means that the ball is going down at the rate of 12 ft/s.

3. Let

$$f(x) = \begin{cases} x^2 - 2x, & x \le 5 \\ 3x, & x > 5 \end{cases}$$

Determine if $f(x)$ is differentiable at $x = 5$. [3]

$$f'_-(5) = \lim_{x\to 5^-} \frac{f(x) - f(5)}{x-5} = \lim_{x\to 5^-} \frac{(x^2 - 2x) - (25 - 10)}{x-5} = \lim_{x\to 5^-} \frac{x^2 - 2x - 15}{(x-5)}$$

$$= \lim_{x\to 5^-} \frac{\cancel{(x-5)}(x+3)}{\cancel{(x-5)}} = 8$$

$$f'_+(5) = \lim_{x\to 5^+} \frac{f(x) - f(5)}{x-5} = \lim_{x\to 5^+} \frac{3x - 3(5)}{x-5} = \lim_{x\to 5^+} \frac{3\cancel{(x-5)}}{\cancel{(x-5)}} = 3$$

Since $f'_-(5) \neq f'_+(5)$, then $f'(5)$ d.n.e.

Therefore, $f(x)$ is NOT differentiable at $x=5$.