

Instructions: Include all relevant work to get full credit.

Quiz #8

1. Find
- $\frac{dy}{dx}$
- by implicit differentiation.

a. $5x^2 - 3y^2 = 10 - 3x\sqrt{y}$

[3]

$$\Rightarrow 10x - 6y \cdot \frac{dy}{dx} = 0 - \left(3x \cdot \frac{1}{2} y^{-1/2} \frac{dy}{dx} + 3\sqrt{y} \right)$$

$$\Rightarrow 10x + 3\sqrt{y} = 6y \frac{dy}{dx} - \frac{3}{2} \frac{x}{\sqrt{y}} \frac{dy}{dx} = \frac{dy}{dx} \left(6y - \frac{3}{2} \frac{x}{\sqrt{y}} \right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{10x + 3\sqrt{y}}{6y - \frac{3}{2} \frac{x}{\sqrt{y}}}$$

b. $\sin(xy) = \cos(x+y)$

[3]

$$\Rightarrow \cos(xy) \cdot \left(x \frac{dy}{dx} + y \right) = -\sin(x+y) \cdot \left(1 + \frac{dy}{dx} \right)$$

$$\Rightarrow x \cos(xy) \cdot \frac{dy}{dx} + \sin(xy) \frac{dy}{dx} = -\sin(x+y) - y \cos(xy)$$

$$\Rightarrow \frac{dy}{dx} = \frac{-\sin(x+y) - y \cos(xy)}{x \cos(xy) + \sin(x+y)}$$

2. Find the derivative of the following functions (You don't have to simplify):

a. $f(x) = \sin^{-1}(3x^2) + \tan^{-1}(1-x)$

[2]

$$\Rightarrow f'(x) = \frac{6x}{\sqrt{1-(3x^2)^2}} + \frac{(-1)}{1+(1-x)^2}$$

b. $g(x) = x^2 \sin^{-1} x + \cos^{-1}(\sin^{-1} x)$

[2]

$$\Rightarrow g'(x) = \frac{x^2}{\sqrt{1-x^2}} + 2x \sin^{-1}(x) + \frac{1}{\sqrt{1-[\sin^{-1}(x)]^2}} \cdot \frac{1}{\sqrt{1-x^2}}$$