NAME Key JULY 9, 2018

Instructions: *Include all relevant work to get full credit.*

Quiz #9

1. Find the derivative of the following functions (*You don't have to simplify*):

a. $f(x) = \log_{10}\sqrt{x^2-1}$ [2]

$= \log_{10}(x^2-1)^{1/2} = \frac{1}{2}\log_{10}(x^2-1)$

$\Rightarrow f'(x) = \frac{1}{2}\cdot\frac{2x}{x^2-1}\cdot\frac{1}{\ln 10}$

b. $g(x) = \ln(x+\ln x^2)$ [2]

$= \ln(x+2\ln x)$

$\Rightarrow g'(x) = \frac{1+2/x}{x+\ln x^2}$

2. Find $\frac{dy}{dx}$ if

a. $y = (\sin x)^{\ln x}$ [2]

$\Rightarrow \ln|y| = \ln\left|(\sin x)^{\ln x}\right| = (\ln x)\ln|\sin(x)|$

$\Rightarrow \frac{y'}{y} = (\ln x)\frac{\cos x}{\sin x} + \frac{1}{x}\ln|\sin x|$

$\Rightarrow y' = (\sin x)^{\ln x}\left[(\ln x)\frac{\cos x}{\sin x} + \frac{1}{x}\ln|\sin x|\right]$

b. $x^y = y^x$ [2]

$\Rightarrow \ln(x^y) = \ln(y^x) \Rightarrow y\ln x = x\ln y$

$\Rightarrow y\left(\frac{1}{x}\right) + y'\ln x = x\frac{1}{y}(y') + (1)\ln y$

$\Rightarrow y'\left(\ln x - \frac{x}{y}\right) = \ln y - \frac{y}{x} \Rightarrow y' = \frac{\ln y - \frac{y}{x}}{\ln x - x/y}$

3. Use logarithmic differentiation to find the derivative of the $y = \frac{\sqrt{x^2+4}\cdot e^{x^2+3x}}{\sqrt[3]{5-2x^3}}$ [2]

$\Rightarrow \ln|y| = \ln\left|\frac{\sqrt{x^2+4}\,e^{x^2+3x}}{\sqrt[3]{5-2x^3}}\right| = \frac{1}{2}\ln|x^2+4| + (x^2+3x) - \frac{1}{3}\ln|5-2x^3|$

$\Rightarrow \frac{y'}{y} = \frac{1}{2}\frac{2x}{(x^2+4)} + (2x+3) - \left(\frac{1}{3}\right)\frac{(-6x^2)}{5-2x^3}$

$\Rightarrow y' = \left(\frac{\sqrt{x^2+4}\,e^{x^2+3x}}{\sqrt[3]{5-2x^3}}\right)\left[\frac{x}{x^2+4} + (2x+3) + \frac{2x^2}{5-2x^3}\right]$