Name Key

Math 245 - Probability and Statistics

September 23, 2014

Long Exam I

Instructions: *Include all relevant work to get full credit. Encircle your final answers.*

1. In a study about truck drivers, the heart rates of 33 sampled drivers were taken. The results of the study are listed in the table below in increasing order.

52	53	55	55	55	57	59	60	60	??	??
??	64	64	66	66	67	68	68	71	71	73
73	74	76	77	77	77	80	??	84	86	91

However, due to sloppy work, the researcher lost 4 of the 33 values. Fortunately, the relative positions of these four values are known and the following summary measures were obtained before the 4 values were lost:

$$\sum_{i=1}^{33} x_i = 2,250 \text{ and } \sum_{i=1}^{33} x_i^2 = 156,702.$$

a. Determine the following:

i. Range. $91-52=39$ [2]

ii. Sample mean. $2250/33 \approx 68.18$ [2]

iii. Standard deviation. 10.14 [5]

$$s^2 = \frac{\sum x^2 - \frac{1}{n}(\sum x)^2}{n-1} = \frac{156702 - \frac{1}{33}(2250^2)}{33-1} = \frac{3292.9}{32} \approx 102.9$$

iv. Median. 67 [2]

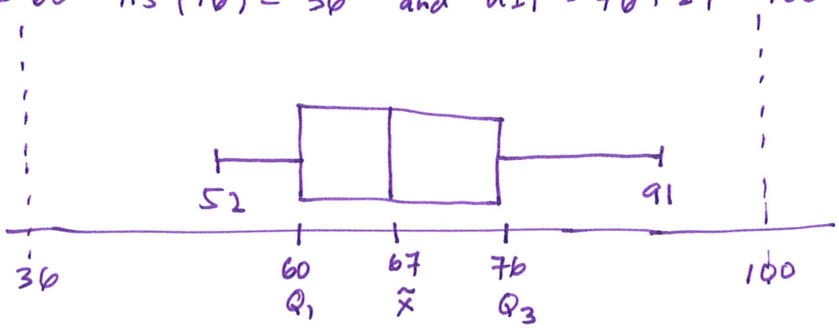
v. The value of first quartile (Q_1). 60 [2]

vi. The value of third quartile (Q_3). 76 [2]

vii. The value of IQR. 16 [2]

b. Draw a (horizontal) **modified** boxplot for this data. Include all relevant values (5-no. summary and inner fences). Are there potential outliers? NO [7]

LIF $= 60 - 1.5(16) = 36$ and UIF $= 76 + 24 = 100$



2. Suppose events A and B are independent with $P(A) = .6$ and $P(B) = .7$ find

a. $P(A|B)$. $= P(A) = .6$ [2]

b. $P(A \cup B)$. $= P(A) + P(B) - P(A \cap B) = .6 + .7 - .42 = .88$ [4]

c. Are events A and B mutually exclusive? Explain. [3]

No, because $P(A \cap B) \neq 0$.

3. If 15 different textbooks (4 English, 5 Mathematics, and 6 History textbooks) were randomly placed on a display shelf, what is the probability that all the English books are together? [4]

$$P(\text{all English books are together}) = \frac{12!\,4!}{15!} \approx .00879$$

4. A certain hospital employs 35 female nurses and 15 male nurses. If a sample of 6 nurses is to be selected to attend a seminar, determine the following:

a. The probability that the 6 nurses selected are all males. *Compute the actual value. Round your answer to 4 decimal places.* [4]

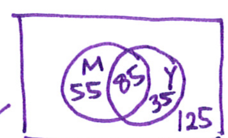
$$\frac{\binom{15}{6}\binom{35}{0}}{\binom{50}{6}} \approx \frac{5005}{15890700} \approx .0003$$

b. The probability that at least one female nurse is selected. *Compute the actual value. Round your answer to 4 decimal places.* [2]

$$P(\text{at least 1 female}) = 1 - P(\text{no female})$$
$$= 1 - .0003 = .9997$$

c. The probability that at least 4 male nurses are selected. *You can leave your answer in combination form.* [4]

$$= \frac{\binom{15}{4}\binom{35}{2} + \binom{15}{5}\binom{35}{1} + \binom{15}{6}\binom{35}{0}}{\binom{50}{6}}$$



5. A recent survey asked 300 people (140 men and 160 women) if they thought women in the the armed forces should be permitted to participate in combat. The results of the survey showed that 40% said 'yes'. Eighty five (85) of those who said 'yes' are men. Suppose a person who participated in this survey is randomly selected. Let M be the event that the person selected is a male, and let Y be the event that the person selected said 'yes'.

$M: 140$
$Y: 120$
$M \cap Y: 85$

a. Using $\cup, \cap$, and ^{c}, express the following probability statements in terms of M and Y. Then compute its value.

i. The probability that the randomly selected person is a male who said 'no'. [4]

$$P(M \cap Y^c) = \frac{55}{300} \approx .18$$

ii. The probability that the randomly selected person is a male or said 'yes'. [4]

$$P(M \cup Y) = P(M) + P(Y) - P(M \cap Y) = \frac{140}{300} + \frac{120}{300} - \frac{85}{300} = \frac{175}{300} \approx .58$$

iii. The probability that the randomly selected person said 'yes' given that he is a male. [4]

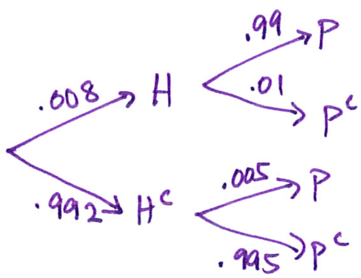
$$P(Y|M) = \frac{P(Y \cap M)}{P(M)} = \frac{85/300}{140/300} = \frac{85}{140} \approx .607$$

b. Are events M and Y independent? Verify using appropriate probabilities. [4]

No, because $P(Y|M) = .607 \neq P(Y) = .4$.

6. In North America, the probability of a person having HIV is .008. A test for HIV yields either a positive or negative result. If a person has HIV, the probability of a positive result is 0.99. If a person does not have HIV, the probability of a positive test result is 0.005. Let H be the event that a randomly selected person from North American has HIV and P be the event that the person is tested positive for HIV. Suppose one person is randomly selected from North America.

a. Construct a tree diagram, showing the different possibilities and their corresponding probabilities. [4]



b. What is the probability that the person doesn't have HIV and will test positive? *Use 5 decimal places.* [2]

$$P(H^c \cap P) = (.992)(.005) = .00496$$

c. What is the probability that the person will test positive? *Use 5 decimal places.* [4]

$$P(P) = (.008)(.99) + (.992)(.005) = .00792 + .00496$$

$$= .01288$$

d. What is the probability that the person doesn't have HIV given that his/her test came out to be positive? *Use 3 decimal places.* [3]

$$P(H^c|P) = \frac{P(H^c \cap P)}{P(P)} = \frac{.00496}{.01288} \approx .385$$

e. Given that the result of the test is negative, what is the probability that the person actually has HIV? *Use 5 decimal places.* [4]

$$P(H|P^c) = \frac{P(H \cap P^c)}{P(P^c)} = \frac{(.008)(.01)}{1-.01288} = \frac{.00008}{.98712} \approx .00008$$

7. The random variable X has the discrete probability distribution shown below.

x	0	1	2	3	4
$P(x)$	?	.20	.25	.30	.10

a. Find $P(X \geq 3)$. [2]

$= P(X=3) + P(X=4) = .30 + .10 \boxed{= .40}$

b. Find $P(X = 0)$. [3]

$= 1 - (.20 + .25 + .30 + .10) = \boxed{.15}$

c. Find the expected value of X. [3]

$E(X) = 0(.15) + 1(.20) + 2(.25) + 3(.30) + 4(.10)$

$= 0 + .2 + .5 + .9 + .4 \boxed{= 2}$

d. Find the expected value of X^2. [3]

$E(X^2) = 0^2(.15) + 1^2(.2) + 2^2(.25) + 3^2(.3) + 4^2(.1)$

$= 0 + .2 + 1 + 2.7 + 1.6$

$\boxed{= 5.5}$

e. Find the standard deviation of X. *Use 3 decimal places.* [4]

$V(X) = E(X^2) - E(X)^2 = 5.5 - (2^2) = 1.5$

$SD(X) = \sqrt{1.5} \approx 1.225$

8. Show that the formula for the sample variance, $S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$, is equivalent to

$S^2 = \frac{1}{n-1}\left(\sum_{i=1}^{n} x_i^2 - \frac{1}{n}(\sum_{i=1}^{n} x_i)^2\right)$ by showing that $\sum_{i=1}^{n}(x_i - \bar{x})^2 = \sum_{i=1}^{n} x_i^2 - \frac{1}{n}\left(\sum_{i=1}^{n} x_i\right)^2$. [7]

Proof: $\sum_{i=1}^{n}(x_i - \bar{x})^2 = \sum_{i=1}^{n}(x_i^2 - 2x_i\bar{x} + \bar{x}^2)$

$= \sum x_i^2 - 2\bar{x}\sum x_i + \sum \bar{x}^2$

$= \sum x_i^2 - 2\bar{x}(n\bar{x}) + n(\bar{x}^2)$

$= (\sum x_i^2) - n\bar{x}^2$

$= \sum x_i^2 - n\left(\frac{\sum x_i}{n}\right)^2$

$= \sum x_i^2 - \frac{1}{n}(\sum x_i)^2$ ∎