Conditional Probability

- **Definition.** For any two events A and B with $P(B) > 0$, the *conditional probability of A given that B has occurred* is defined by

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Example 1. In a small town of 2000 people, there are 800 males, 700 of whom are employed. If a total 250 people are unemployed in this town, find the probability that a randomly selected person is

1. a male?
2. employed?
3. an employed male?
4. employed given he is a male?
5. male given the person is employed?

Example 2. Suppose that of all individuals buying a certain digital camera, 60% include an optional memory card in their purchase, 40% include an extra battery, and 30% include both a card and battery.

1. Given that the selected individual purchased an extra battery, what is the probability that an optional memory card was also purchased?

2. What is the probability that an extra battery is included in the purchase given that the person got the optional memory card?

- **Independence of Events.** Events A and B are said to be *independent* if one of the following is true:

1. $P(A|B) = P(A)$
2. $P(B|A) = P(B)$
3. $P(A \cap B) = P(A) \cdot P(B)$

- **Independence of More Then Two Events.** Events $A_1, \ldots, A_n$ are *mutually independent* if for every k ($k = 2, 3, \ldots, n$) and every subset of indices $i_1, i_2, \ldots, i_k$,

$$P(A_{i_1} \cap A_{i_2} \cap \cdots \cap A_{i_k}) = P(A_{i_1}) \cdot P(A_{i_2}) \cdot \cdots \cdot P(A_{i_k})$$

Example. Do #75 on page 87

- **The Multiplication Rule.**

$$P(A \cap B) = P(A|B) \cdot P(B) \quad \text{or} \quad P(A \cap B) = P(B|A) \cdot P(A)$$

- **Total Probability.** Let $A_1, \ldots, A_k$ be mutually exclusive and complementary events (That is, $A_1, \ldots, A_k$ form a partition of the sample space). Then for any other event B,

$$\begin{aligned} P(B) &= P(B \cap A_1) + P(B \cap A_2) + \cdots + P(B \cap A_k) \\ &= P(B|A_1)P(A_1) + P(B|A_2)P(A_2) + \cdots + P(B|A_k)P(A_k) \end{aligned}$$

- **Bayes' Rule.** Let $A_1, \ldots, A_k$ be a collection of k mutually exclusive and complementary events with $P(A_i) > 0$ for $i = 1, \ldots, k$. Then for any other event B for which $P(B) > 0$,

$$P(A_j|B) = \frac{P(A_j \cap B)}{P(B)} = \frac{P(B|A_j)P(A_j)}{\sum_{i=1}^{k} P(B|A_i)P(A_i)}$$

Example 3. A chain of video stores sells three different brands of DVDs. Of its DVDs sales, 50% are brand 1 (the least expensive), 30% are brand 2, and 20% are brand 3. Each manufacturer offers a 1-year warranty on parts and labor. It is known that 25% of brand 1's DVDs require warranty repair work, whereas the corresponding percentages for brands 2 and 3 are 20% and 10%, respectively.

1. What is the probability that a randomly selected purchaser bought a brand 1 DVD that will need repair while under warranty?

2. What is the probability that a randomly selected purchaser bought a DVD that will need repair while under warranty?

3. If a customer returns to the store with a DVD that needs warranty repair work, what is the probability that it is a brand 1 DVD?

Practice. Online chat rooms are dominated by the young. Teens are the biggest users. If we look only at adult internet users (age 18 and over), 47% of the 18 to 29 age group chat, as do 21% of those aged 30 to 49 and just 7% of those 50 and over. It is known that 29% of adult internet users are age 18 to 29, another 47% are 30 to 49 years old, and the remaining 24% are age 50 and over. If an adult internet user is randomly selected, what is the probability that

1. the person is at least 50 years old?

2. the person chats online given he/she is at least 50 years old?

3. the person is at least 50 years old and chats online?

4. the person chats online?

5. the person is at least 50 years old given that the person chats online?

- **Homework problems:**
Sec 2.4 (pp. 80-83) # 45, 47, 51, 53, 59, 61, 63, 67.
Sec 2.5 (pp. 86-88) # 71, 73, 77, 79, 85, 87.