Continuous Distributions.

- **Continuous R.V.** A random variable X is said to be *continuous* if its set of possible values is an entire interval of numbers – that is, if for some $A < B$, any number x between A and B is possible.

- **Continuous Probability Distribution.** Let X be a continuous random variable. Then a *probability distribution* or *probability density function* (pdf) of X is a function $f(x)$ such that for any two numbers a and b with $a \leq b$,

$$P(a \leq X \leq b) = \int_a^b f(x)\,dx$$

That is, the probability that X takes on a value in the interval $[a, b]$ is the area above this interval and under the graph of the density function.

- **Example 1.** "Time headway" in traffic flow is the elapsed time between the time that one car finishes passing a fixed point and the instant that the next car begins to pass that point. Let $X =$ the time headway for two randomly chosen consecutive cars on a freeway during a period of heavy flow. The following pdf of X is essentially the one suggested in "The Statistical Properties of Freeway Traffice" (*Transp. Research, vol. 11:221-228*):

$$f(x) = 0.15e^{-0.15(x-.5)}, \qquad x \geq .5$$

and 0 elsewhere.

  1. Sketch the graph of $f(x)$.
  2. Verify that the total area under the graph equals to 1.
  3. Compute $P(X \leq 5)$.
  4. Compute $P(X = 5)$.
  5. Compute $P(5 < X < 8)$.

- **Uniform Distribution.** A Continuous r.v. X is said to have a *uniform distribution* on the interval $[A, B]$ if the pdf of X is

$$f(x) = \frac{1}{B - A}, \qquad A \leq X \leq B$$

and 0 elsewhere.

- **Example 2.** Consider a r.v. $X \sim Unif[2, 8]$. Find $P(3.5 < X < 5.5)$.

- **Cumulative Distribution.** The *cumulative distribution function* $F(X)$ for a continuous r.v. X is defined for every number x by

$$F(x) = P(X \leq x) = \int_{-\infty}^{x} f(y)\,dy$$

For each x, $F(x)$ is the area under the density curve to the left of x.

- **Mean and Variance.** Let X be a random variable with pdf $f(x)$, then

  1. $E(X) = \int_{-\infty}^{\infty} xf(x)\,dx \qquad (= \mu)$
  2. $E[h(X)] = \int_{-\infty}^{\infty} h(x)f(x)\,dx$
  3. $Var(X) = E[(X - \mu)^2] = \int_{-\infty}^{\infty} (x - \mu)^2 f(x)\,dx \qquad (= \sigma^2)$

- **Homework problems:**
  Section 4.1: pp. 142-143; # 1, 3, 5, 7, 9.
  Section 4.2: pp. 150-152; # 11, 13, 21, 23.