Counting Techniques

- **Equally Likely Model.** If an experiment has n different possible outcomes each of which has the same chance of occurring, then the probability that a specific outcome will occur is $1/n$. If an event occurs in k out of these n outcomes, then the probability of the event A, $P(A)$, is

$$P(A) = \frac{\text{no. of favorable outcomes}}{\text{no. of possible outcomes}} = \frac{k}{n}$$

  Examples:

  1. If a card is drawn from a well-shuffled standard deck of cards, what is the probability of getting

  a. a heart?

  b. a face card?

  c. a heart or a face card?

  2. If two cards are drawn, what is the probability of getting a pair (two of a kind)?

  3. If five cards are drawn, what is the probability of getting a flush (all cards of the same suit)?

- **Fundamental Principle of Counting (Product Rule).** Suppose two tasks are to be performed in succession. If the first task can be done in exactly n_1 different ways, and the second task can be done independently in n_2 different ways, then the sequence of things can be done in $n_1 \times n_2$ different ways.

  Examples:

  1. If you roll a six-sided die and then pick a card from the standard deck of cards, how many different possible outcomes are there?

  2. In a certain town there are 3 male and 2 female candidates for mayor and 4 male and 2 female candidates for vice-mayor. If each candidate has the same chance of winning the election, what is the probability that after the election they will have a female mayor and a male vice-mayor?

- **General Fundamental Principle of Counting.** Suppose k tasks are to be performed in succession. If the first task can be done in exactly n_1 different ways, the second task can be done independently in n_2 different ways, the third task can be done independently in n_3 different ways, and so forth, then this sequence of k tasks can be done in $n_1 n_2 n_3 \cdots n_k$ different ways.

  Examples:

  1. In a statistical study, an individual is classified according to gender, income bracket (upper, middle, or lower class) and highest level of educational attained (elementary, high school, or college). Find the number of ways in which an individual can be classified.

  2. Eight elementary students are to be lined up to board a bus. How many different possible arrangements are there?

  3. Three girls and nine boys, including Mark and Jim, are to be lined up to get in a bus.

  a. In how many ways can this be done?

  b. In how many ways can this be done if all the girls insist to be together?

  c. In how many ways can this be done if Mark and Jim refused to be together?

4. Suppose there are 3 male and 2 female professors, 7 male and 2 female associate professors, and 4 male and 2 female assistant professors in the Mathematics department. A committee consisting of a professor, an associate professor, and an assistant professor is to be set up to review the current math curriculum of the department. Assuming that each faculty member has the same chance of being selected for the committee work, what is the probability that the committee will be composed of all female teachers?

- **Permutation.** Any ordered sequence of k objects taken from a set of n distinct objects is called a *permutation* of size k of the objects. The number of permutations of size k that can be constructed from the n objects is denoted by $P(n,k)$,

$$P(n,k) = n(n-1)(n-2)\cdots(n-k+2)(n-k+1) = \frac{n!}{(n-k)!}$$

Examples:

1. Compute $P(5,2)$, $P(5,3)$, and $P(5,5)$.

2. In a local swimming competition, there are 20 contestants. The first, second, and third placer will be awarded with gold, silver, and bronze medals, respectively. How many different possible competition results are there?

3. There are 10 teaching assistants available for grading papers in a statistics course at a large university. The first exam consists of four questions, and the professor wishes to select a different assistant to grade each question (only one assistant per question). In how many ways can the assistant be chosen for grading?

4. How many different ways can you sit 12 students in a round table?

5. How many different "words" can be made using all the letters of the word STATISTICS?

6. How many different ways can you distribute 20 candies to 5 kids? What if you have to give at least one candy to each kid?

- **Combination.** Given a set of n distinct objects, any unordered subset of size k of the objects is called a *combination*. The number of combinations of size k that can be formed from n distinct objects will be denoted by $\binom{n}{k}$.

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

Examples:

1. Compute $\binom{5}{2}$, $\binom{5}{3}$, $\binom{5}{5}$.

2. Prove that $\binom{n}{k} = \binom{n}{n-k}$.

3. A jar contains 6 white and 4 red marbles. If 3 marbles are randomly selected, what is the probability of selecting

a. two white and one red marbles?

b. all white marbles?

c. all of the same colors?

4. If you draw 5 cards from a standard deck of cards, how many different possible outcomes are there?

5. If 5 cards are randomly selected from a well-shuffled standard deck of cards, what is the probability that selected cards will form a

a. flush (all of the same suit).

b. full house (a trio and a pair).

6. There are 22 male and 9 female professors in the Mathematics department at UW-L. A committee of 5 members is to be formed to review the current math curriculum of the department. If each professor has equal chance of being selected for this committee, what is the probability that the committee will have a female majority?

- **Homework.**

Sec 2.3: (pp. 65-67) 29, 31, 33, 35, 37, 39, 41, 43.