Common Discrete Distributions.

- **Binomial Distribution.**

  1. *Binomial Random Variable.*

  a. Consider a Bernoulli trial with success probability p.

  b. Perform this Bernoulli trial n times.

  c. Let X represent the number of successes out of the n trials.

  2. *Binomial Probability Distribution.* $X \sim b(x; n, p)$

  $$p(x) = \binom{n}{x} p^x (1-p)^{n-x} \qquad x = 0, 1, \ldots, n$$

  and 0 elsewhere.

  3. *Properties.*

  a. $E(X) = np$.

  b. $Var(X) = np(1-p)$.

- **Hypergeometric Distribution.**

  1. *Hypergeometric Random Variable.* Suppose a sample of n objects is randomly selected (without replacement) from a population of N objects containing M successes. If X denote the number of successes out of the n selected objects, then X follows the *hypergeometric distribution.*

  2. *Hypergeometric Probability Distribution.* $X \sim h(x; n, M, N)$

  $$p(x) = \frac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}}$$

  for integer x satisfying $\max(0, n-N+M) \le x \le \min(n, M)$.

  3. *Properties.*

  a. $E(X) = n\left(\frac{M}{N}\right)$.

  b. $Var(X) = \left(\frac{N-n}{N-1}\right) n \left(\frac{M}{N}\right)\left(1 - \frac{M}{N}\right)$.

- **Negative Binomial Distribution.**

  1. *Negative Binomial Random Variable.*

  a. Consider a Bernoulli trial with success probability p.

  b. Perform this Bernoulli trial until a total of $r > 0$ successes have been observed.

  c. Let X represent the number of failures that precede the rth success.

  2. *Negative Binomial Probability Distribution.* $X \sim nb(x; r, p)$

  $$p(x) = \binom{x+r-1}{r-1} p^r (1-p)^x \qquad x = 0, 1, 2, \ldots$$

  and 0 elsewhere.

  3. *Properties.*

  a. $E(X) = \dfrac{r(1-p)}{p}$.

  b. $Var(X) = \dfrac{r(1-p)}{p^2}$.

- **Homework problems:**
  Section 3.4: pp. 120-122; # 47, 49, 55, 57, 59, 65.
  Section 3.5: pp. 127-128; # 69, 71, 75, 77.

- **Poisson Experiments.** Experiments yielding numerical values of a random variable X, the number of outcomes occurring during a given interval or in a specified region, are called *Poisson experiments.*

  The given interval may be of any length, such as a minute, a day, a week, a month, or even a year. Hence, the Poisson experiment can generate observations for the random variable X representing the number of telephone calls per hour received by a office, the number of days school is closed due to snow during the winter, or the number of traffic accidents in La Crosse in January.

- **Poisson Process.**

  1. The number of outcomes occurring in one time interval or specified region is independent of the number that occurs in any other disjoint time interval or region of space. *In this way, we say that the Poisson process has no memory.*

  2. The probability that a single outcome will occur during a very short time interval or in a small region is proportional to the length of the time interval or the size of the region and does not depend on the number of outcomes occurring outside this time interval or region.

  3. The probability that more than one outcome will occur in such a short time interval or fall in such a small region is negligible.

- **Poisson Probability Distribution.** A random variable X is said to have a *Poisson distribution* with parameter $\lambda(\lambda > 0)$ if the pmf of X is

$$p(x;\lambda) = \frac{e^{-\lambda}\lambda^x}{x!} \qquad x = 0, 1, 2, \ldots$$

- **Examples.** Do # 80, #84, and #86 on page 132.

- If X has a Poisson distribution with parameter λ, then $E(X) = V(X) = \lambda$.

  Proof:

- **Binomial Approximation.** Suppose that in the binomial pmf $b(x;n,p)$, we let $n \to \infty$ and $p \to 0$ in such a way that np approaches a value $\lambda > 0$. Then $b(x;n,p) \to p(x;\lambda)$.

- **Example.** Do #88 on page 132.

- **Homework problems:**

  Section 3.6: pp. 132-133; # 79, 81, 83, 85, 87, 89, 91.