Expected Value of R.V.

- **Discrete Random Variable:** A *discrete random variable* X has a countable number of possible values (That implies that there is a gap between possible values).
- **Discrete Probability Distribution:** Consider a discrete random variable X with only k possible outcomes. The table below is a discrete probability distribution of X if the following two requirements are satisfied:

  1. Every probability p_i is a number between 0 and 1.

  2. $p_1 + p_2 + \cdots + p_k = 1$

x	x_1	x_2	x_3	$\cdots$	x_k
$\Pr(X = x)$	p_1	p_2	p_3	$\cdots$	p_k

- **Expected Value (or Mean).** Let X be a discrete r.v. with set of possible values D and pmf $p(x)$. The *expected value* or *mean value* of X, denoted by $E(X)$ or μ_X, is

$$E(X) = \mu_X = \sum_{x \in D} x \cdot p(x)$$

When the r.v. has a finite possible values, the **Mean** (or **expected** value) of X is

$$\mu = E(X) = x_1 p_1 + x_2 p_2 + \cdots + x_k p_k = \sum_{i=1}^{k} x_i p_i$$

Example 1. Let X be the number of cars sold by Mike on a typical Saturday. Based on past experience, the probability of selling x cars is given below

x	0	1	2	3	4
$P(X = x)$	.20	.40	.25	0.10	0.05

Determine the expected number of cars the Mike will be able to sell this coming Saturday.

Example 2. Determine the expected value of a Bernoulli random variable X, if the probability of success is p.

Example 3. Consider the experiment of continually rolling a fair die until the first "5" results. Let X denote the number of trials needed to get this first "5".

1. Determine the probability distribution for X.

2. Show that the sum of the probabilities for all possible values of X is 1.

3. Determine the expected value of this random variable.

- **The Expected Value of a Function of R.V.** If the r.v. X has a set of possible values D and pmf $p(x)$, then the expected value of any function $h(X)$, denoted by $E[h(X)]$ or $\mu_{h(X)}$, is computed by

$$E[h(X)] = \sum_{D} h(x) \cdot p(x)$$

Example 4. In example 1, if Mike earns a fix salary of \$50 per day and gets a commission of \$250 per car sold, determine his expected income on a typical Saturday.

- **Proposition.** $E(aX + b) = a \cdot E(X) + b$

proof:

- **Variance of R.V.** Let X have pmf $p(x)$ and expected value μ. Then the *variance* of X, denoted by $V(X)$ or σ_X^2, is

$$V(X) = \sum_{x \in D} (x - \mu)^2 \cdot p(x) = E[(X - \mu)^2].$$

The *standard deviation* (SD) of X is $\sigma_X = \sqrt{\sigma_X^2}$

Example 5. In example 1, determine the standard deviation of X.

- Show that $V(X) = \left[\sum_{D} x^2 \cdot p(x)\right] - \mu^2 = E(X^2) - [E(X)]^2$.

- Show that $V(aX + b) = a^2 V(X)$.

- **Homework problems:**
  Section 3.3: pp. 113-114; # 29, 33, 37, 41.