Gamma Distribution

- **Gamma Function.** For $\alpha > 0$, the *gamma function* $\Gamma(\alpha)$ is defined by

$$\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} e^{-x}\, dx$$

- **Gamma Distribution.** A continuous random variable X is said to have a *gamma distribution* with parameters α and β, where $\alpha > 0, \beta > 0$, if the pdf of X is

$$f(x) = \frac{1}{\beta^\alpha \Gamma(\alpha)} x^{\alpha-1} e^{-x/\beta} \qquad x \geq 0$$

  Remarks:

  1. $E(X) = \alpha\beta$.

  2. $Var(X) = \alpha\beta^2$.

- **Special Cases:**

  1. *Standard Gamma Distribution.* When $\beta = 1$.
  A continuous random variable X is said to have a *standard gamma distribution* if the pdf of X is

$$f(x) = \frac{1}{\Gamma(\alpha)} x^{\alpha-1} e^{-x} \qquad x \geq 0$$

  Remarks:

  a. $E(X) =$

  b. $Var(X) =$

  2. *Exponential Distribution.* When $\alpha = 1$ and $\beta = \frac{1}{\lambda}$.
  A continuous random variable X is said to have an *exponential distribution* with parameter $\alpha > 0$ if the pdf of X is

$$f(x) = \lambda e^{-\lambda x} \qquad x \geq 0$$

  Remarks:

  a. $E(X) =$

  b. $Var(X) =$

  3. *Chi-Squared Distribution.* When $\alpha = \nu/2$ and $\beta = 2$.
  A continuous random variable X is said to have a *chi-squared distribution* with parameter $\nu > 0$ if the pdf of X is

$$f(x) = \frac{1}{2^{\nu/2}\Gamma(\nu/2)} x^{\nu/2-1} e^{-x/2} \qquad x \geq 0$$

  The parameter ν is called the *number of degrees of freedom* (df) of X. The symbol χ^2 is often used in place of "chi-squared".
  Remarks:

  a. $E(X) =$

  b. $Var(X) =$

- **Homework problems:**
  Section 4.4: pp. 170-171; # 59, 61, 63, 65, 67.