Joint Distributions

- **Two Discrete Random Variables.** Let X and Y be two discrete r.v.'s defined on the sample space S of an experiment. The *joint probability mass function* $p(x, y)$ is defined for each pair of numbers (x, y) by
$$p(x, y) = P(X = x \text{ and } Y = y)$$
Let A be any set consisting of pairs of (x, y) values. Then the probability $P[(X, Y) \in A]$ is obtained by summing the joint pmf over pairs in A:
$$P[(X, Y) \in A] = \sum \sum_{(x,y)\in A} p(x, y)$$
Example 5.1 (on page 194)

- **Marginal Distributions.** The *marginal probability mass functions* of X and of Y, denoted by $p_X(x)$ and $p_Y(y)$, are given by
$$p_X(x) = \sum_y p(x, y) \qquad \text{and} \qquad p_Y(y) = \sum_x p(x, y)$$
Example 5.1 continued

- **Two Continuous Random Variables.** Let X and Y be two continuous r.v.'s. Then $f(x, y)$ is the *joint probability density function* for X and Y if for any two-dimensional set A
$$P[(X, Y) \in A] = \int_A \int f(x, y) dy dx$$
In particular, if A is a two-dimensional rectangle $\{(x, y) : a \leq x \leq b, c \leq y \leq d\}$, then
$$P[(X, Y) \in A] = P(a \leq x \leq b, c \leq y \leq d) = \int_a^b \int_c^d f(x, y) dy dx$$
Example 5.3 (on page 196)

- **Marginal Density.** The *marginal probability density functions* of X and Y, denoted by $f_X(x)$ and $f_Y(y)$, respectively, are given by

$$f_X(x) = \int_{-\infty}^{\infty} f(x,y)\,dy \qquad \text{and} \qquad f_Y(y) = \int_{-\infty}^{\infty} f(x,y)\,dx$$

Example 5.3 continued

Example 5.5 (on page 197)

- **Independence.** Two random variables X and Y are said to be *independent* if for every pair of x and y values,

$$p(x,y) = p_X(x)p_Y(y)$$

or

$$f(x,y) = f_X(x)f_Y(y)$$

Otherwise, X and Y are said to be *dependent.*

Example 5.8 (on page 200)

- **Homework problems:**
Section 5.1: pp. 203-206; # 1, 3, 7, 9, 13, 19.