Normal Distribution

- **Normal Distribution.** A continuous r.v. X is said to have a *normal distribution* with mean μ and standard deviation σ (or μ and σ^2), where $-\infty < \mu < \infty$ and $\sigma > 0$, if the pdf of X is

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2\sigma^2}(x-\mu)^2} \qquad -\infty < x < \infty$$

Example: Too much cholesterol in the blood increases the risk of heart disease. Young women are generally less afflicted with high cholesterol than other groups. The cholesterol levels of women aged 20 to 34 follow an approximately normal distribution with mean 185 milligrams per deciliter (mg/dl) and standard deviation 39 mg/dl.

1. What percent of young women have cholesterol levels below 245 mg/dl? *Sketch an appropriate normal curve and shade the area under the curve that corresponds to this question.*

2. What percent of young women have cholesterol levels above 245 mg/dl? *Sketch an appropriate normal curve and shade the area under the curve that corresponds to this question.*

3. What percent of young women have cholesterol levels between 210 and 245 mg/dl? *Sketch an appropriate normal curve and shade the area under the curve that corresponds to this question.*

4. What cholesterol value will put a woman in the extreme 5% of the population?

- **Standard Normal Distribution.** The normal distribution with parameter values $\mu = 0$ and $\sigma = 1$ is called the *standard normal distribution.* The standard normal distribution random variable is usually denoted by Z and its pdf is given by

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2} \qquad -\infty < z < \infty$$

  Determine the following probabilities

  1. $P(Z < 1.96)$
  2. $P(|Z| < 1.96)$
  3. $P(Z <$ _____ $) = 0.95$

- **Notations.**

  1. $\Phi(z) = P(Z < z)$ is the standard normal cdf.
  2. z_α denotes the value on the measurement axis for which α of the area under the z curve lies to the right of z_α. That is, $\alpha = 1 - \Phi(z_\alpha)$.

- **Z-score.** If $X \sim N(x; \mu, \sigma)$, then $Z = \dfrac{X - \mu}{\sigma} \sim N(0, 1)$.

- **Practice.** Do # 34 on page 162 and 46 on page 163

- **Binomial Approximation.** If $X \sim b(x; n, p)$, n is large, and p is not too close to 0 or 1, then $X \approx N(x; \mu = np, \sigma = \sqrt{np(1-p)})$.

  Note: *When a continuous distribution is used to approximate a discrete r.v., a correction may improve the approximation.*

  1. $P(X \leq x) = \Phi\left(\dfrac{x + .5 - np}{\sqrt{np(1-p)}}\right)$
  2. $P(X < x) = \Phi\left(\dfrac{x - .5 - np}{\sqrt{np(1-p)}}\right)$

- **Example.** Do #54 on page 164

- **Homework problems:**
  Section 4.3: pp. 162-164; # 29, 31, 33, 35, 37,53,55.