Nonlinear Regression - Exponential

- **Nonlinear Regression Models.**

$$Y_i = f(\boldsymbol{X}_i, \boldsymbol{\gamma}) + \epsilon_i$$

- **Exponential Regression.**

  1. $Y_i = \gamma_0 e^{\gamma_1 X_i} + \epsilon_i$, where $\epsilon_i \sim N(0, \sigma^2)$

  2. $Y_i = \gamma_0 + \gamma_1 e^{\gamma_2 X_i} + \epsilon_i$, where $\epsilon_i \sim N(0, \sigma^2)$

  3. $Y_i = \gamma_0 + \gamma_1 X_{i1} + \gamma_3 e^{\gamma_2 X_{i2}} + \epsilon_i$, where $\epsilon_i \sim N(0, \sigma^2)$

```
# Example of Type I
curve(5*exp(x/2),0,10)

x=sample(1:10,100,replace=T)
y=5*exp(x/2)+rnorm(100,mean=0,sd=20)
plot(x,y)
results=nls(y~b*exp(a*x),start=list(a=1,b=3))
coef(results)
#         a         b
# 0.4820824 5.9215071

curve(5.9215*exp(x*0.482),0,10,add=T,col="darkred",lwd=2)

# Example of Type II
curve(100-50*exp(-2*x),0,2)

x2=runif(100,0,2)
y2=100-50*exp(-2*x2)+rnorm(100,mean=0,sd=2)
plot(x2,y2)

results2=nls(y2~a+b*exp(c*x2),start=list(a=1,b=-1,c=-1))
round(coef(results2),2)
#      a      b      c
# 100.32 -50.05  -1.98

curve(100.32-50.05*exp(x*(-1.98)),0,2,add=T,col="darkred",lwd=2)
```

- **Example.** A hospital administrator wished to develop a regression model for predicting the degree of long-term recovery after discharge from the hospital for severely injured patients. The predictor variable to be utilized is number of days of hospitalization (X), and the response variable is a prognostic index for long-term recovery (Y), with large values of the index reflecting a good prognosis. The data are stored in the file 'InjuredPatients.csv'.

```
data=read.csv("InjuredPatients.csv",header=T)
attach(data)
Y=index; X=days
results3=nls(Y~b*exp(a*X),start=list(a=0,b=50))

plot(X,Y,xlab="Days Hospitalized",ylab="Prognostic Index")
curve(58.60653*exp(-0.03959*x),0,70,add=T,col="darkred",lwd=2)

Y.fitted=58.60653*exp(-0.03959*X)
residuals=Y-Y.fitted
plot(Y.fitted,residuals,main="Residual Plot")
```

```
# Or you can also use the 'glm' function
log.lin <- glm(Y~X, family=quasi(link="log", variance="constant"))
coef(log.lin)
# (Intercept)           X
#  4.07084672 -0.03958645     # Note: exp(4.07084672)=58.60656
```

- **Logistic Regression Models.**

$$Y_i = \frac{\gamma_0}{1+\gamma_1 e^{(\gamma_2 X_i)}} + \epsilon_i$$

```
# Logistic Regression Models

curve(10/(1+20*exp(-2*x)),0,4,lwd=2)

x=runif(100,0,5)
y=10/(1+20*exp(-2*x))+rnorm(100,mean=0,sd=.5)
plot(x,y)

nls(y~c/(1+b*exp(a*x)),start=list(a=-2,b=20,c=10))
#    data: parent.frame()
#      a      b      c
# -1.997 20.380  9.963
curve(9.963/(1+20.38*exp(-1.997*x)),0,5,col="darkred",lwd=2,add=T)


# Problem 13.10: Enzyme kinetics

data.kinetics=read.csv("Kinetics.csv",header=T)
attach(data.kinetics)
y=velocity; x=concentration
y.p=1/y
x.p=1/x
results=lm(y.p~x.p) #bo=0.03376 and b1=0.45401

nls(y~a*x/(b+x),start=list(a=1/0.03376,b=0.454/0.03376))
plot(concentration,velocity)
curve(28.14*x/(12.57+x),0,40,add=T,lwd=2)

y.fitted=28.14*x/(12.57+x)
residuals=y-y.fitted
plot(y.fitted,residuals)
```